جَامعة
المَنارة
MANARA UNIVERSITY

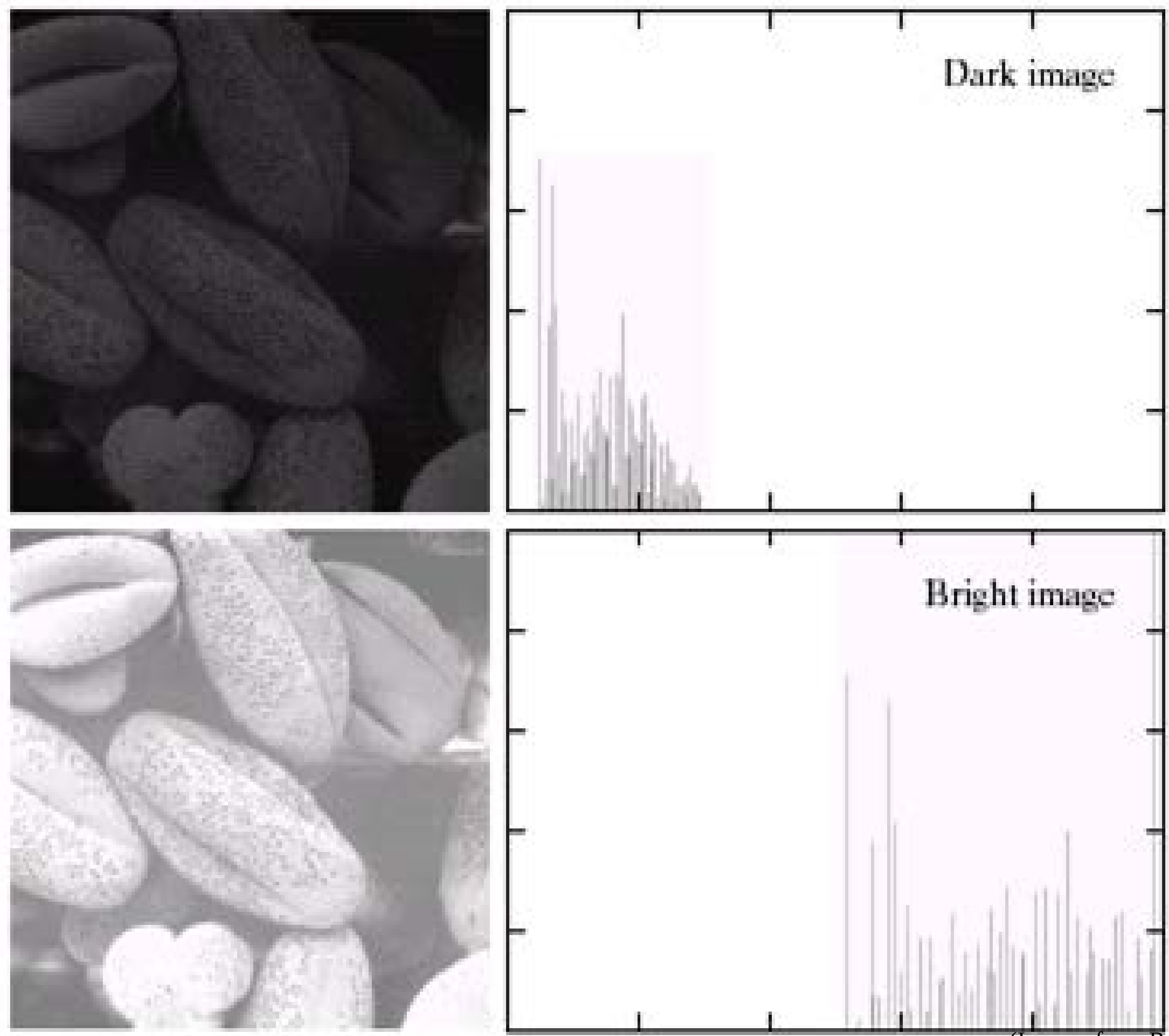
DIGITAL IMAGE PROCESSING

CHAPTER 3 - 2

IMAGE ENHANCEMENT IN THE SPATIAL DOMAIN

- The histogram of a digital image with gray levels from 0 to L-1 is a discrete function $h(r_k)=n_k$, where:
  - r_k is the k^{th} gray level
  - n_k is the # pixels in the image with that gray level
  - n is the total number of pixels in the image
  - k = 0, 1, 2, …, L-1
- Normalized histogram: $p(r_k)=n_k/n$
  - sum of all components = 1

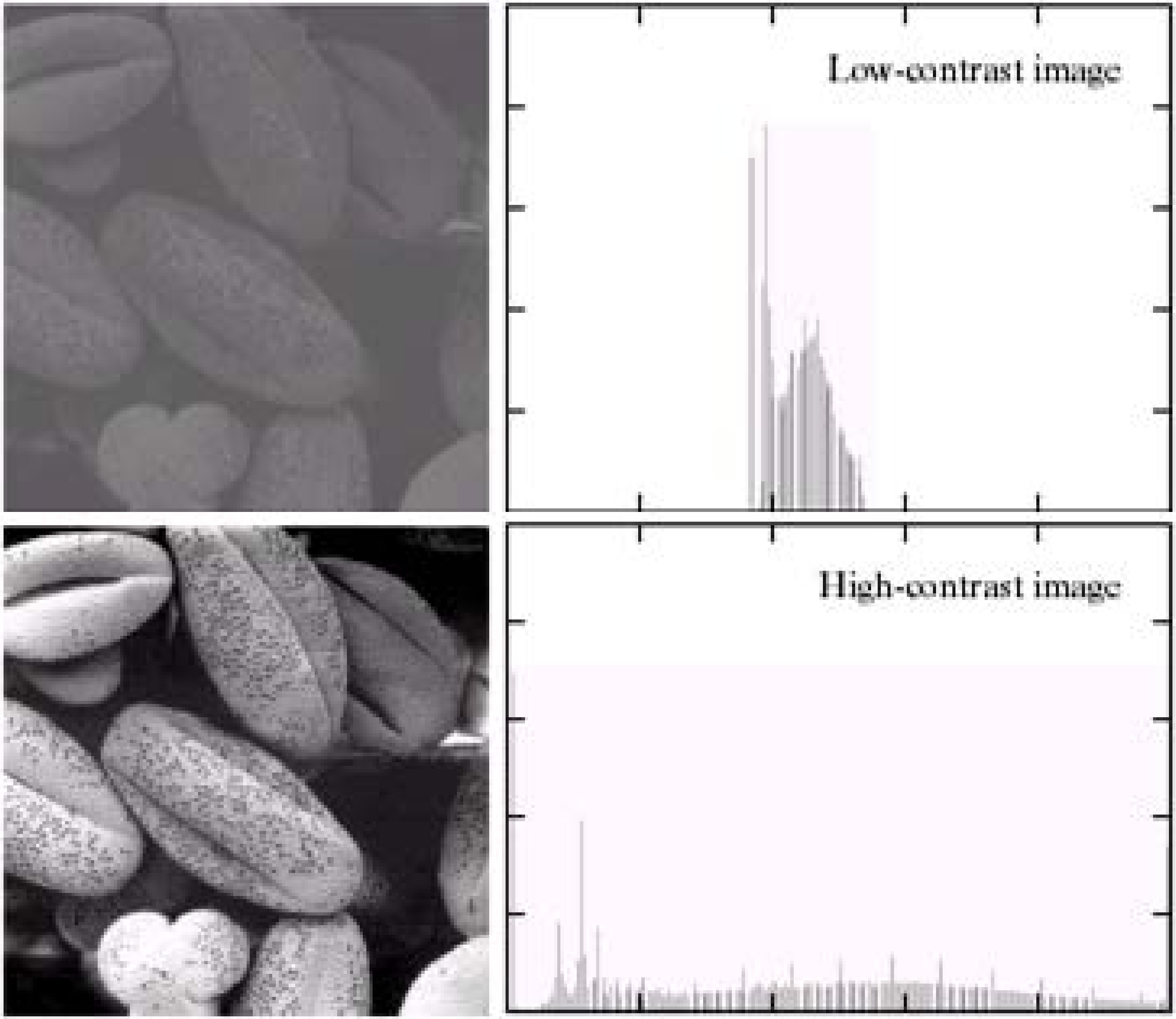
Histogram of an Image



https://manara.edu.sy/

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Histogram of an Image (cont.)



https://manara.edu.sy/

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Histogram Processing

- The shape of the histogram of an image does provide useful info about the possibility for contrast enhancement.

- Types of processing:

  Histogram equalization
  Histogram matching (specification)
  Local enhancement

Histogram Equalization

- As mentioned above, for gray levels that take on discrete values, we deal with probabilities:

$$p_r(r_k)=n_k/n,\ k=0,1,..,\ L-1$$

- The plot of $p_r(r_k)$ versus r_k is called a histogram and the technique used for obtaining a uniform histogram is known as histogram equalization (or histogram linearization).

Histogram Equalization

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j)$$

$$= \sum_{j=0}^{k} \frac{n_j}{N}$$

- Histogram equalization(HE) results are similar to contrast stretching but offer the advantage of full automation, since HE automatically determines a transformation function to produce a new image with a uniform histogram.

Histogram Equalization Algorithm

- The intermediate steps of the histogram equalization process are:

- Take the cumulative histogram of the image to be equalized
- Normalize the cumulative histogram to 255 (L-1)
- Use the normalized cumulative histogram as the mapping function of the original image

```
totalPixels = row*col;
greyValue   = image[i][j];
pixels = 0;
for (int k=0; k<(greyValue+1); k++)
  pixels = pixels + histgram[k];
equalizedImage[i][j] = pixels/totalPixels*(L-1);
```

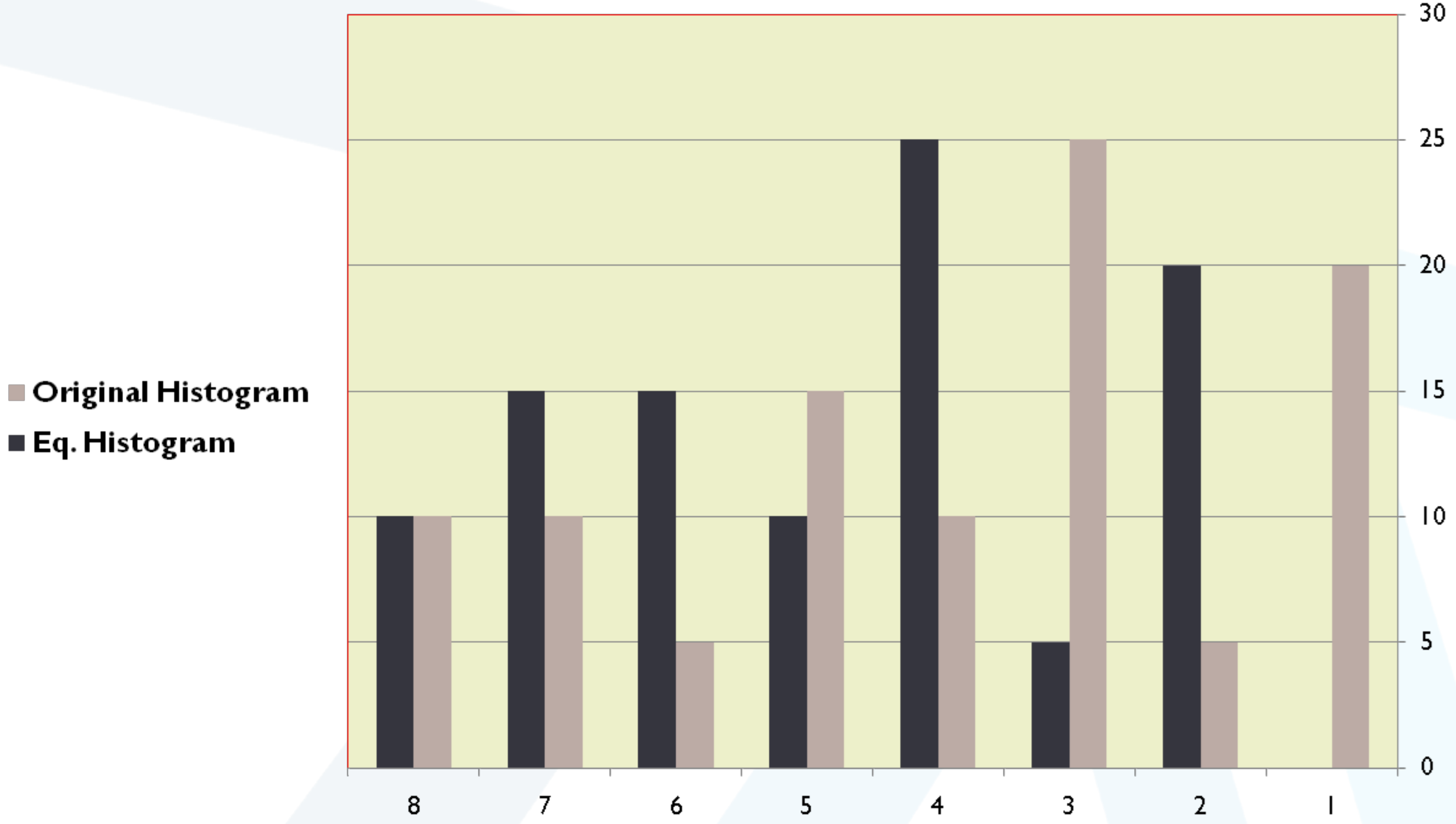
Histogram Equalization Example

Intensity	# pixels
0	20
1	5
2	25
3	10
4	15
5	5
6	10
7	10
Total	100

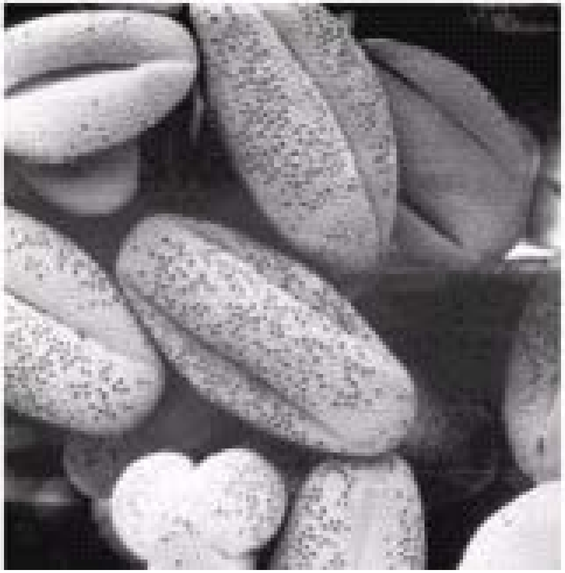
Accumulative Sum of P_r
20/100 = 0.2
(20+5)/100 = 0.25
(20+5+25)/100 = 0.5
(20+5+25+10)/100 = 0.6
(20+5+25+10+15)/100 = 0.75
(20+5+25+10+15+5)/100 = 0.8
(20+5+25+10+15+5+10)/100 = 0.9
(20+5+25+10+15+5+10+10)/100 = 1.0
1.0

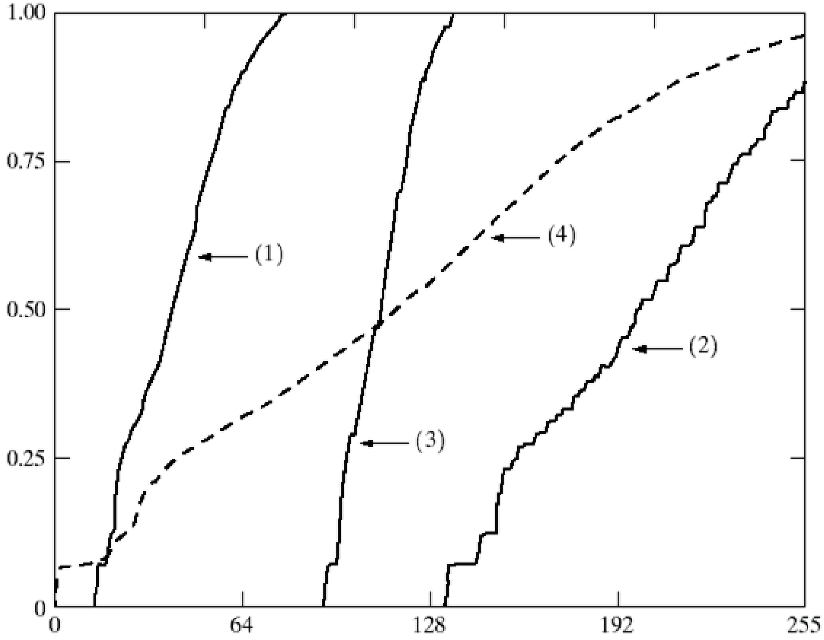
Histogram Equalization Example (cont.)

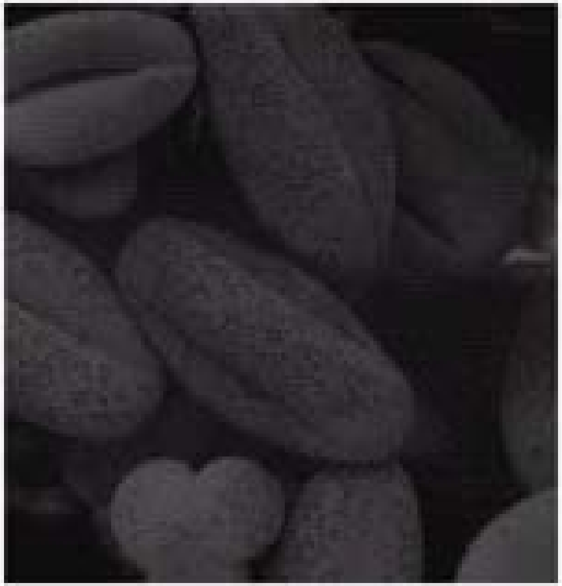
Intensity (r)	No. of Pixels (n_j)	Acc Sum of P_r	Output value	Quantized Output (s)
0	20	0.2	0.2x7 = 1.4	1
1	5	0.25	0.25*7 = 1.75	2
2	25	0.5	0.5*7 = 3.5	3
3	10	0.6	0.6*7 = 4.2	4
4	15	0.75	0.75*7 = 5.25	5
5	5	0.8	0.8*7 = 5.6	6
6	10	0.9	0.9*7 = 6.3	6
7	10	1.0	1.0x7 = 7	7
Total	100			

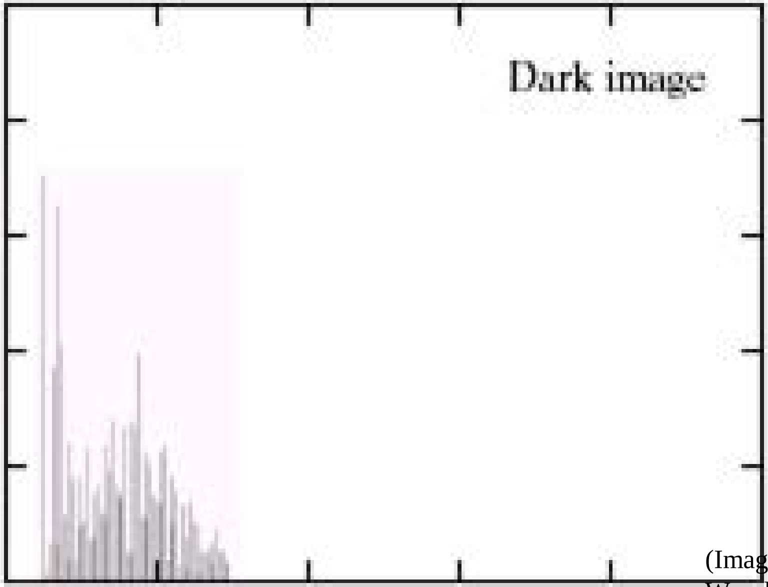

Original Histogram
Eq. Histogram
30
25
20
15
10
5
0
8
7
6
5
4
3
2
1

Histogram Equalization



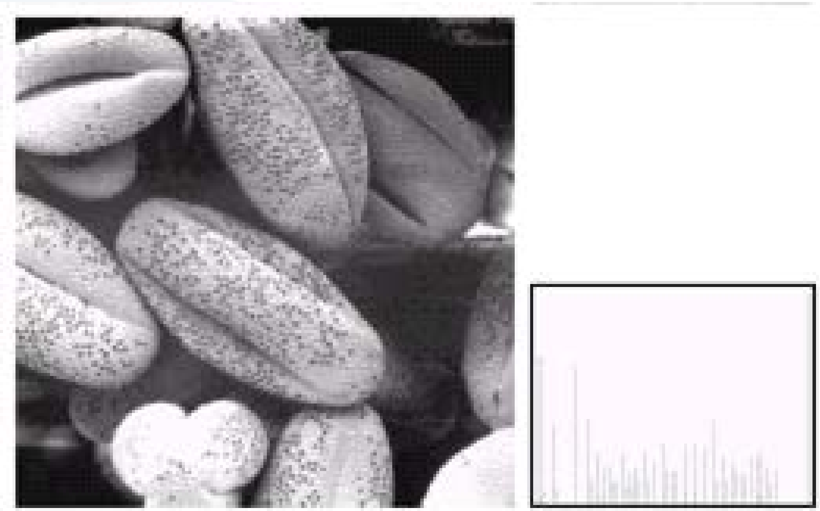


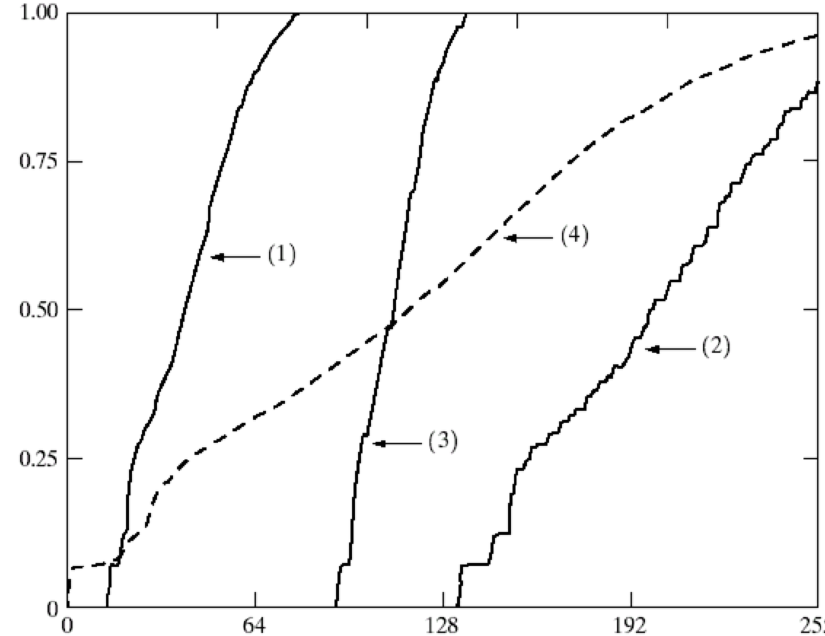


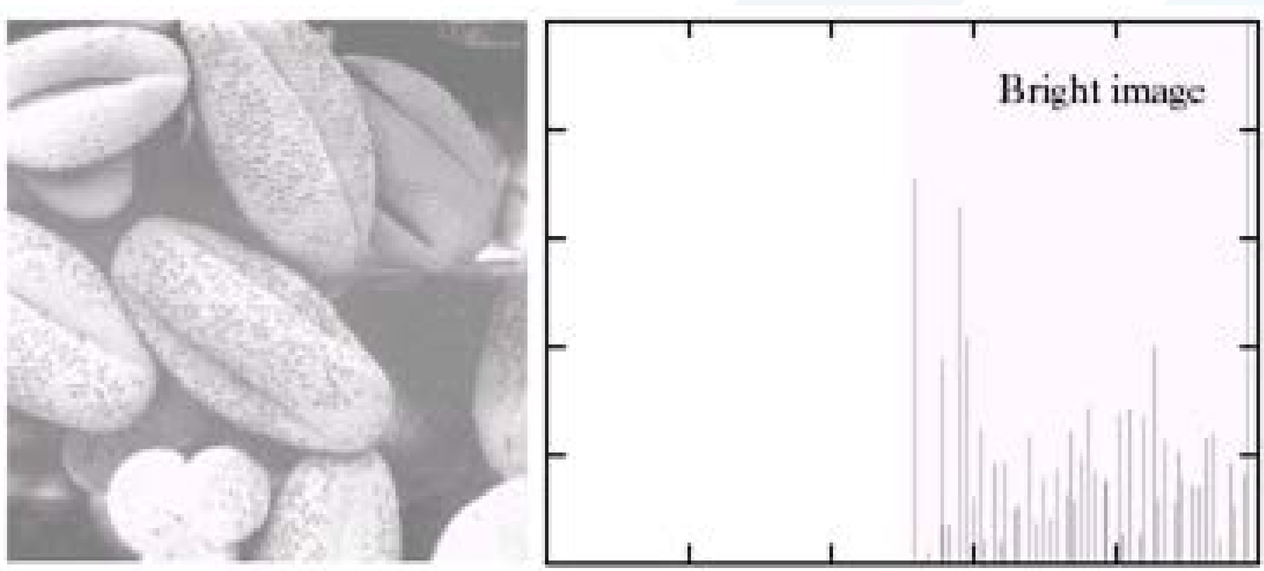


(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Histogram Equalization (cont.)

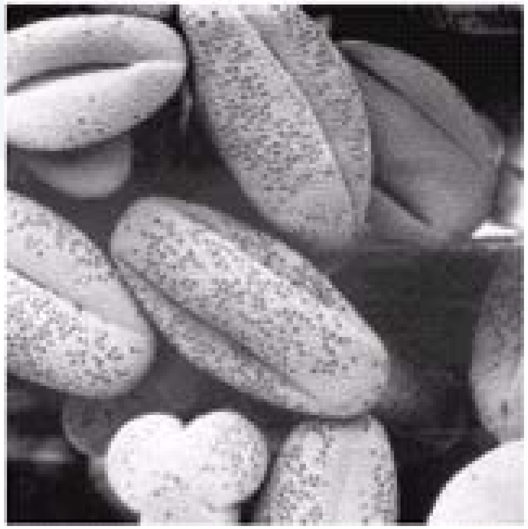


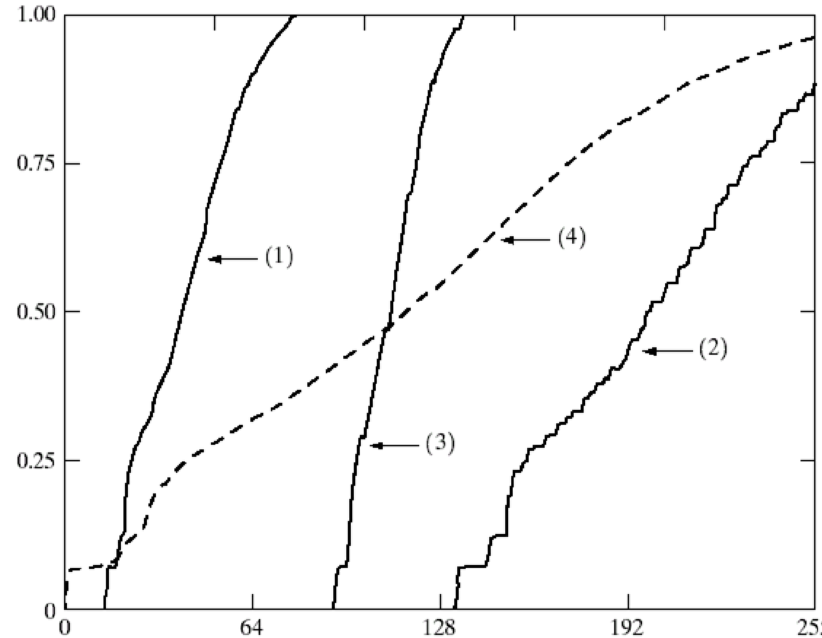


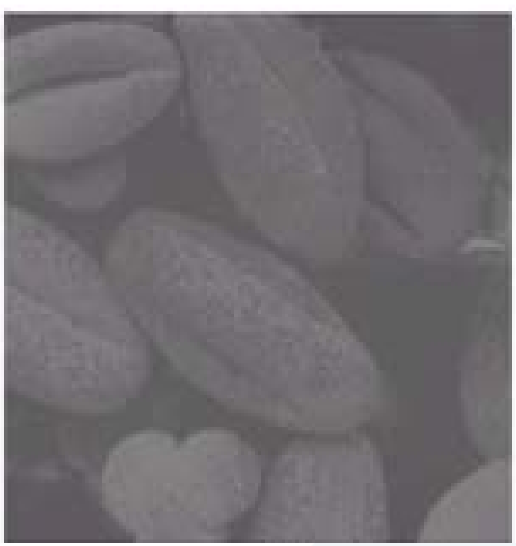


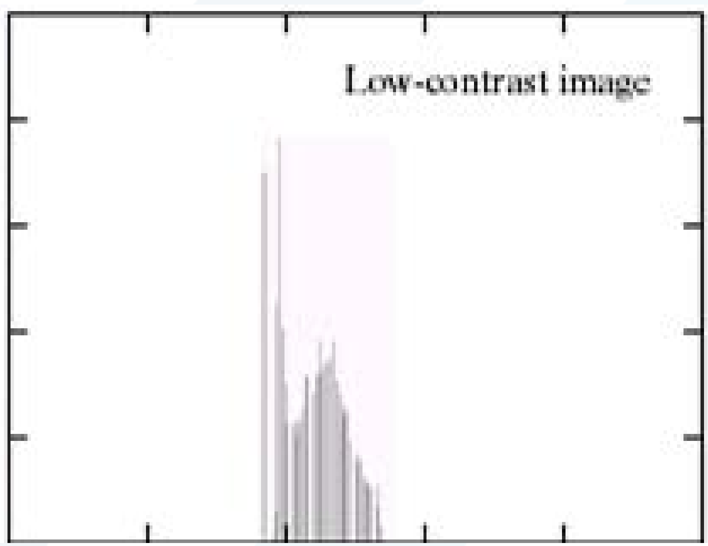
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Histogram Equalization (cont.)



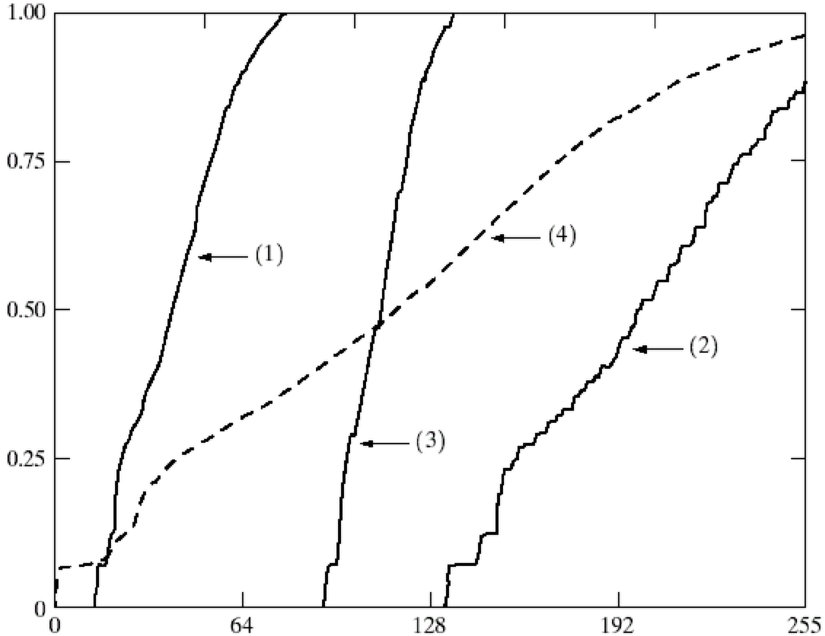


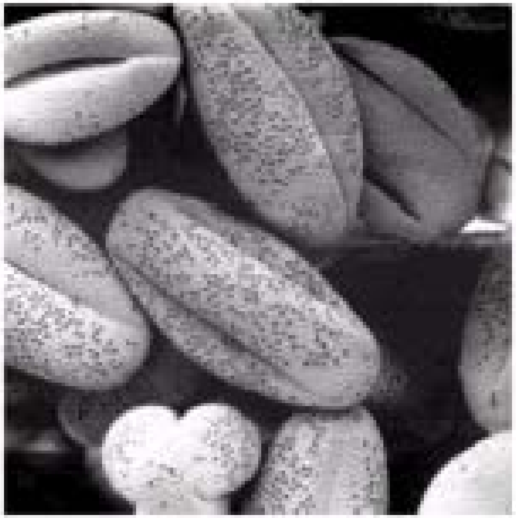


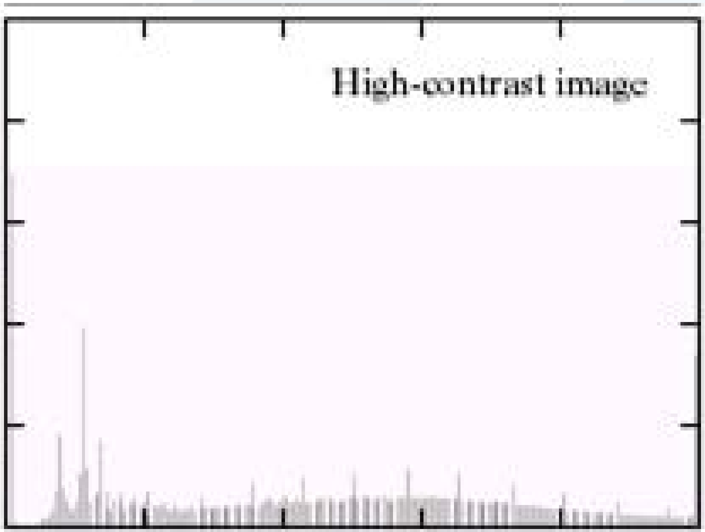


(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Histogram Equalization (cont.)

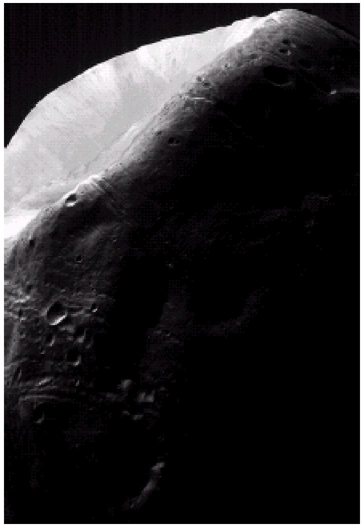


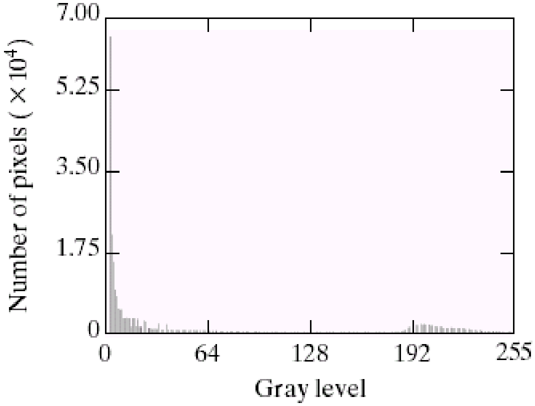


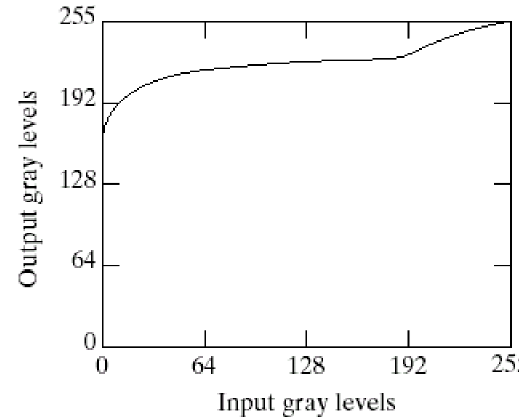


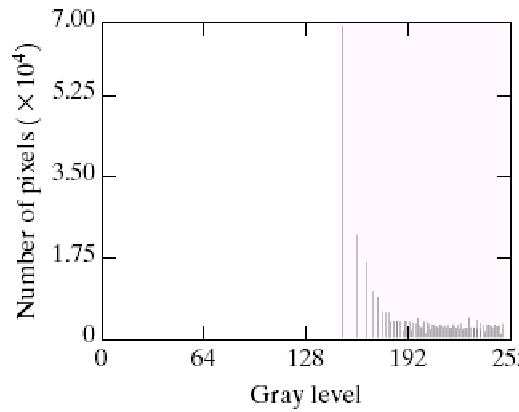
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Histogram Equalization (cont.)









Original image

After Histogram Eq.

Problem: Histogram equalization gives an image with very Low Contrast

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Histogram Matching (or Specification)

- Histogram equalization does not allow interactive image enhancement and generates only one result: an approximation to a uniform histogram.

- Sometimes though, we need to be able to specify particular histogram shapes capable of highlighting certain gray-level ranges.

Histogram Matching : Algorithm

The procedure for histogram-matching based enhancement is:

- ❑ Equalize the levels of the original image using:

$$s = T(r_k) = \sum_{j=0}^{k} \frac{n_j}{n}$$

n: total number of pixels,

n_j: number of pixels with gray level r_j,

$k = 0, 1, \ldots, L-1$

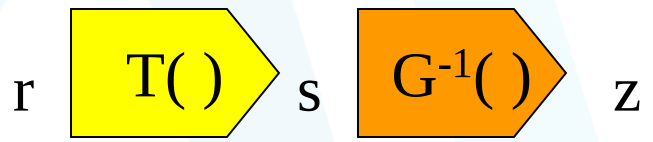
L: number of discrete gray levels

- ❑ Specify the desired density function and obtain the transformation function G(z):

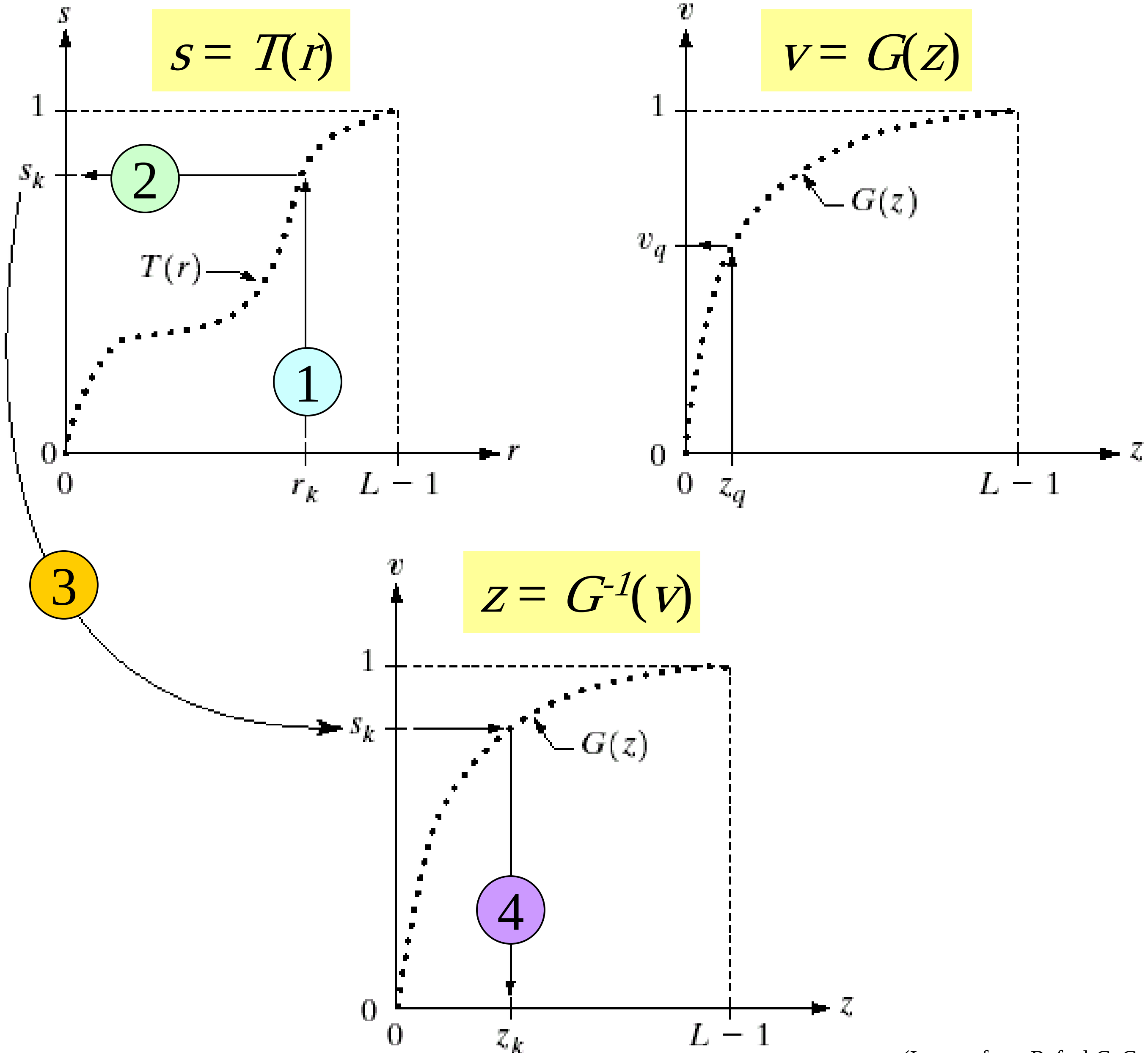
$$v = G(z) = \sum_{0}^{z} p_z(w) \approx \sum_{i=0}^{z} \frac{n_i}{n}$$

p_z: specified desirable PDF for output

- ❑ Apply the inverse transformation function $z = G^{-1}(s)$ to the levels obtained in step 1.



Histogram Matching : Algorithm (cont.)



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Histogram Specification

- The principal difficulty in applying the histogram specification method to image enhancement lies in being able to construct a meaningful histogram. So…

- Either a particular probability density function (such as a Gaussian density) is specified and then a histogram is formed by digitizing the given function,

- Or a histogram shape is specified on a graphic device and then is fed into the processor executing the histogram specification algorithm.

Histogram Matching Example

Histogram of input image

Intensity (s)	# pixels
0	20
1	5
2	25
3	10
4	15
5	5
6	10
7	10
Total	100

Original data

Need to do Histogram of output image

Intensity (z)	# pixels
0	5
1	10
2	15
3	20
4	20
5	15
6	10
7	5
Total	100

User defined

Histogram Matching Example (cont.)

1. build Histogram Equalization for both tables

r	(n_j)	ΣP_r	s
0	20	0.2	1
1	5	0.25	2
2	25	0.5	3
3	10	0.6	4
4	15	0.75	5
5	5	0.8	6
6	10	0.9	6
7	10	1.0	7

$$s_k = T(r_k)$$

z	(n_j)	ΣP_z	v
0	5	0.05	0
1	10	0.15	1
2	15	0.3	2
3	20	0.5	4
4	20	0.7	5
5	15	0.85	6
6	10	0.95	7
7	5	1.0	7

$$v_k = G(z_k)$$

Histogram Matching Example (cont.)

2. Make the table Map

r → s

r	s
0	1
1	2
2	3
3	4
4	5
5	6
6	6
7	7

s → v

v → z

v	z
0	0
1	1
2	2
4	3
5	4
6	5
7	6
7	7

result

r	z
0	1
1	2
2	2
3	3
4	4
5	5
6	5
7	6

Actual Output Histogram

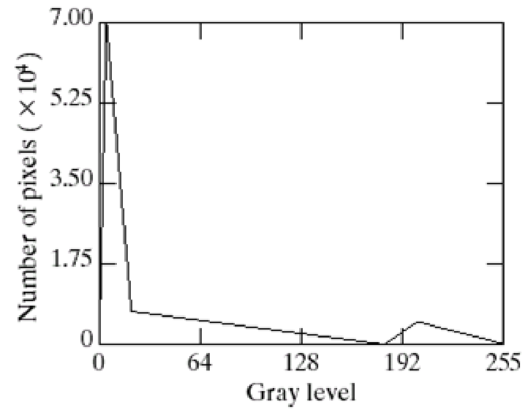
z	# Pixels
0	0
1	20
2	30
3	10
4	15
5	15
6	10
7	0

$s_k = T(r_k)$

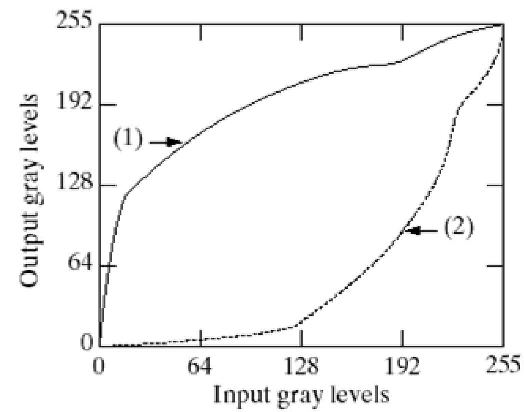
$z_k = G^{-1}(v_k)$

Histogram Matching Example (cont.)

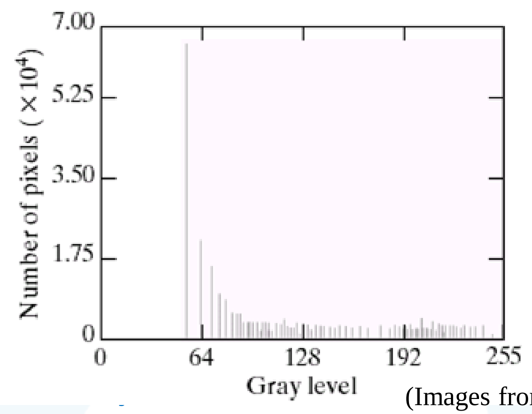
Desired histogram



Transfer function

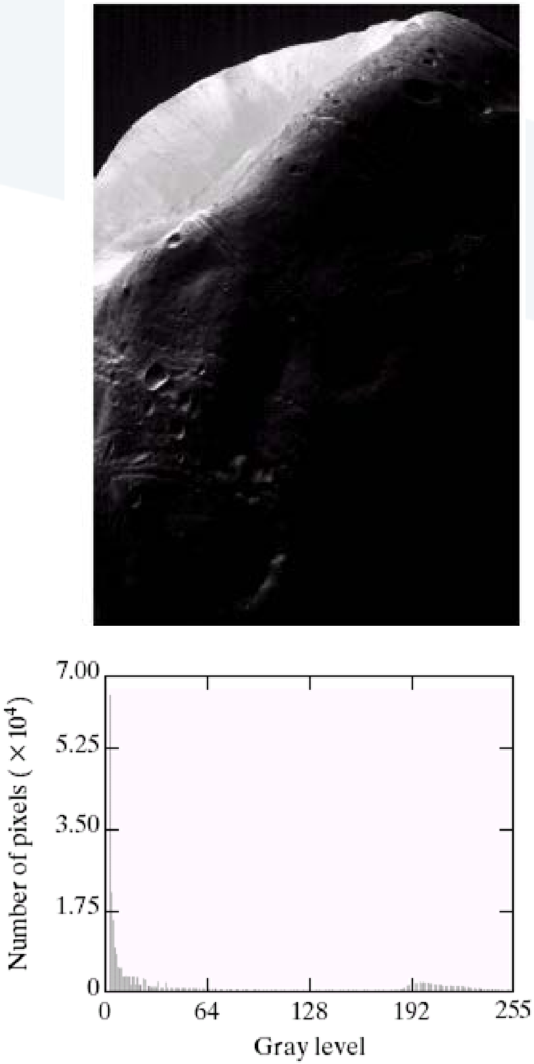


Actual histogram

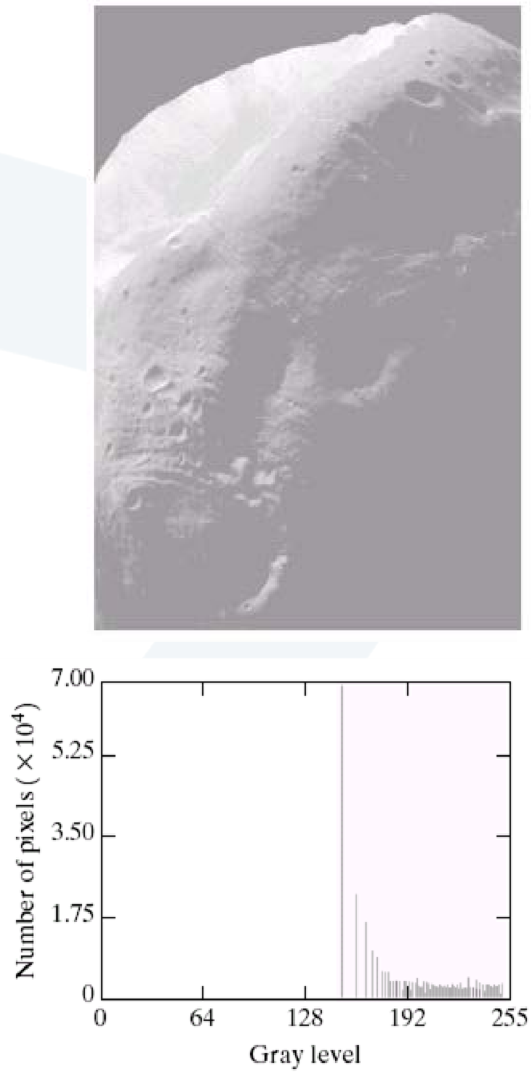


(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

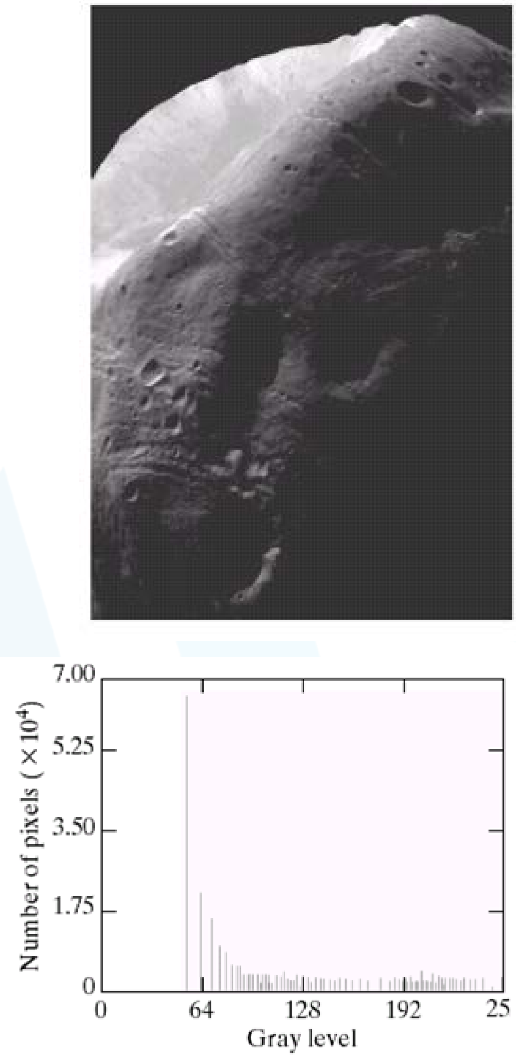
Histogram Matching Example (cont.)



Original image



After histogram equalization

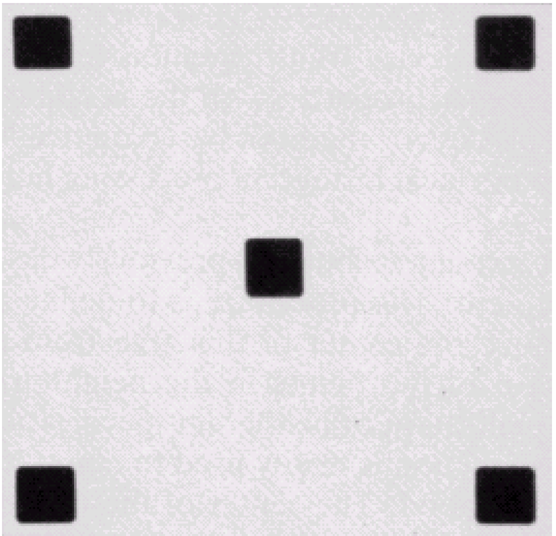


After histogram matching

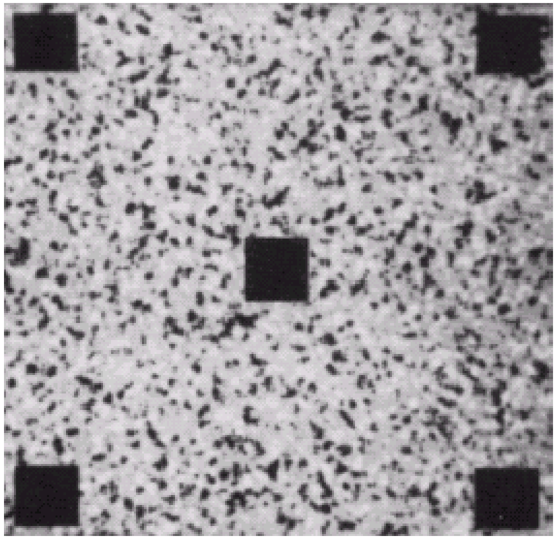
Local Enhancement : Local Histogram Equalization

Concept: Perform histogram equalization in a small neighborhood

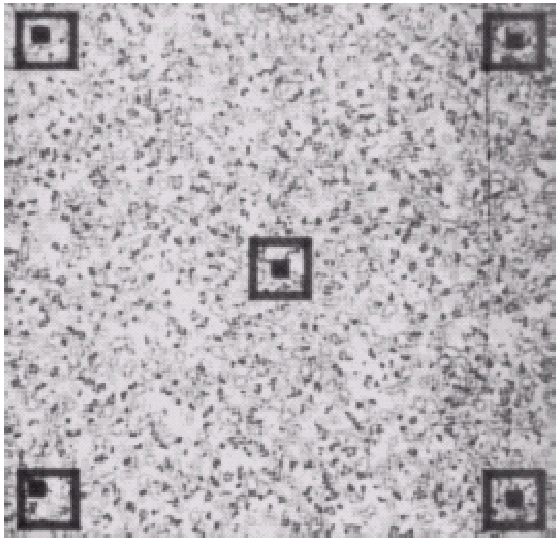
Orignal image



After Hist Eq.



After Local Hist Eq.
In 7x7 neighborhood



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Local Enhancement : Histogram Statistic for Image Enhancement

We can calculate statistical values such as Mean, Variance of Local area

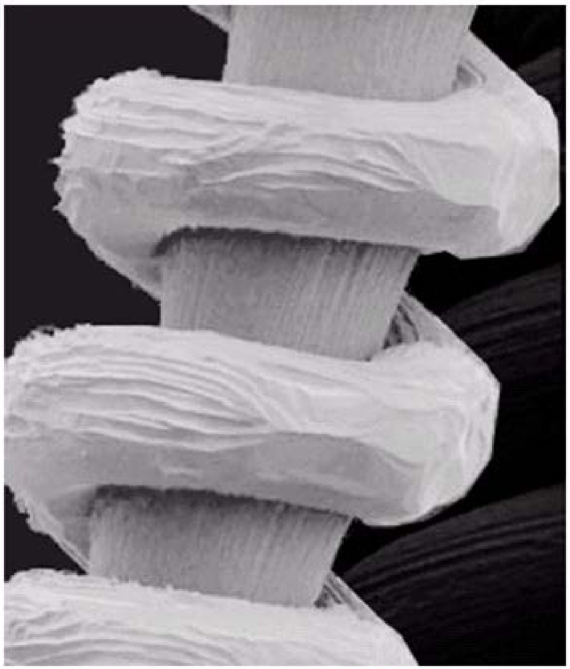


Image of filament taken by electronic camera

The right corner of the image contain another filament (dark)

We need to increase the brightness of the background

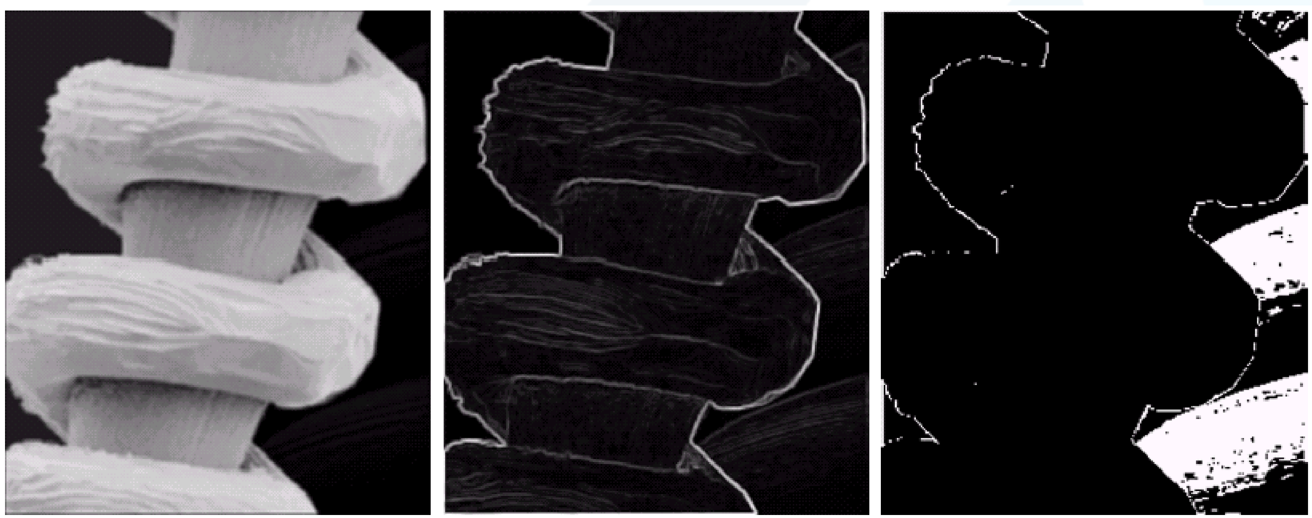
Adjust the brightness of entire image

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Sample equations for Local enhancement (specific for this task)

$$g(x,y) = \begin{cases} E \cdot f(x,y) \; when \; m_{s_{xy}} \leq k_0 M_G \; and k_1 D_G \sigma_{s_{xy}} k_2 M_G \\ f(x,y) \; otherwise \end{cases}$$

Original image | Local Variance image | Multiplication factor



a b c

FIGURE 3.25 (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Local Enhancement

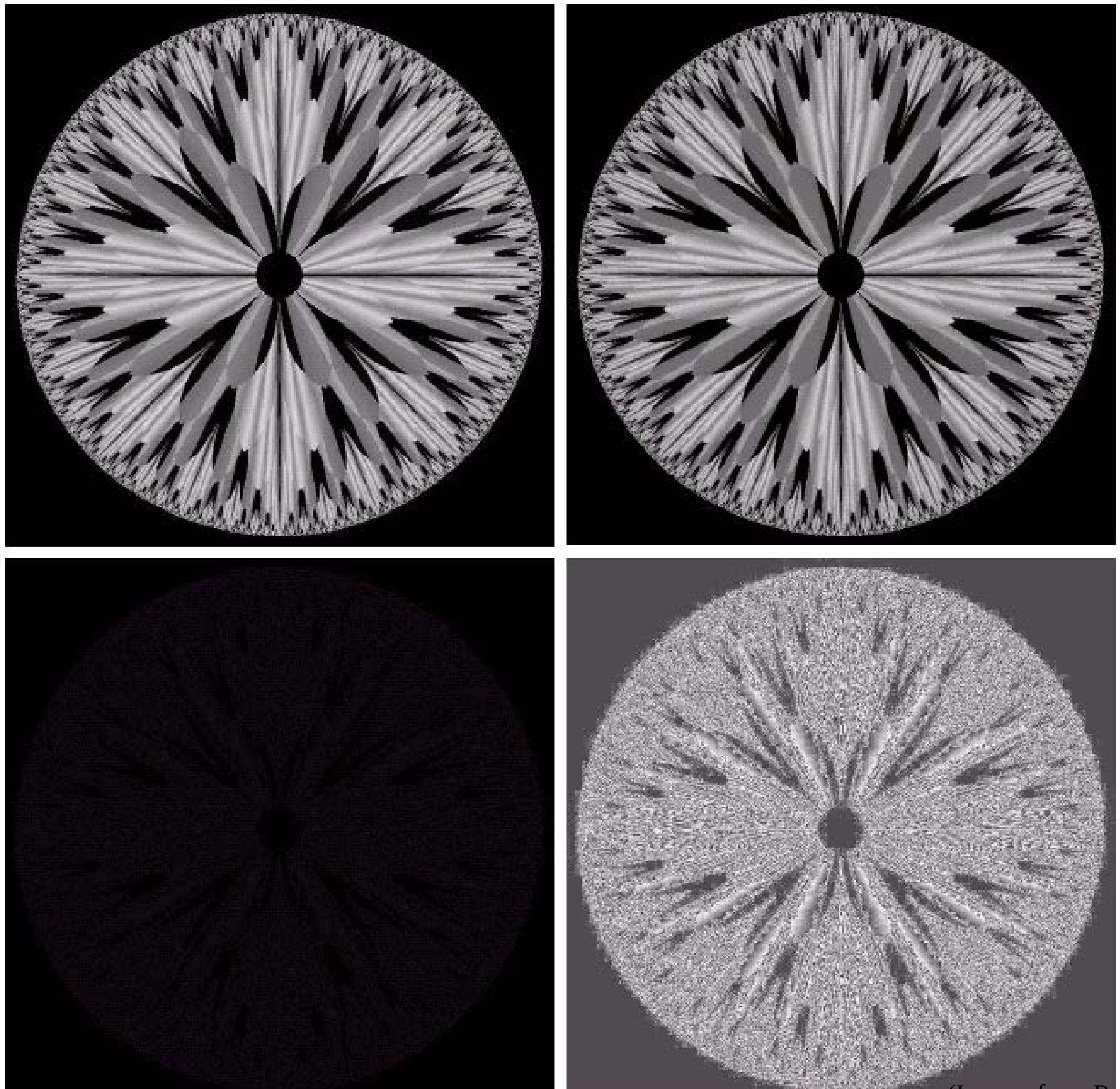
FIGURE 3.26 Enhanced SEM image. Compare with Fig. 3.24. Note in particular the enhanced area on the right side of the image.

Output image

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Arithmetic Operation: Subtraction

Application: Error measurement



a b
c d

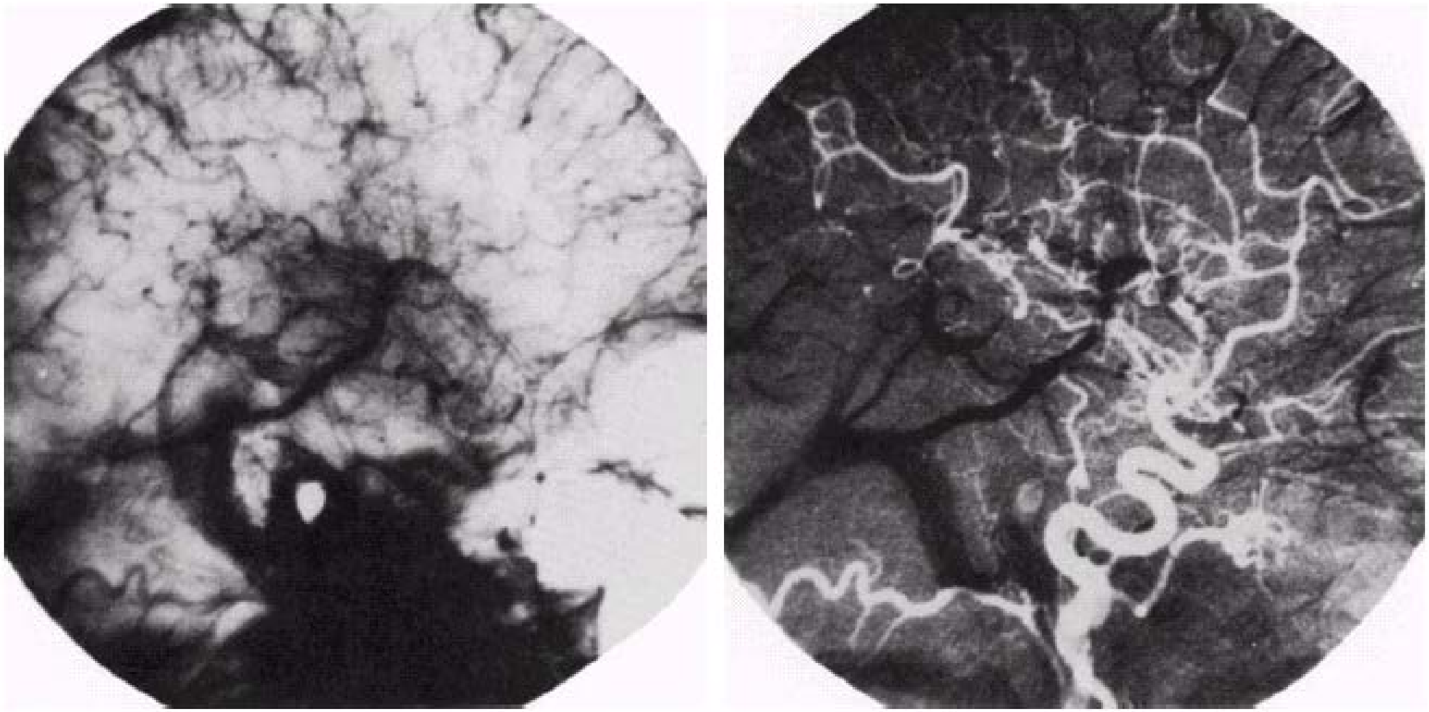
FIGURE 3.28 (a) Original fractal image. (b) Result of setting the four lower-order bit planes to zero. (c) Difference between (a) and (b). (d) Histogram-equalized difference image. (Original image courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA).

Error image

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Arithmetic Operation: Subtraction (cont.)

Application: Mask mode radiography in angiography work

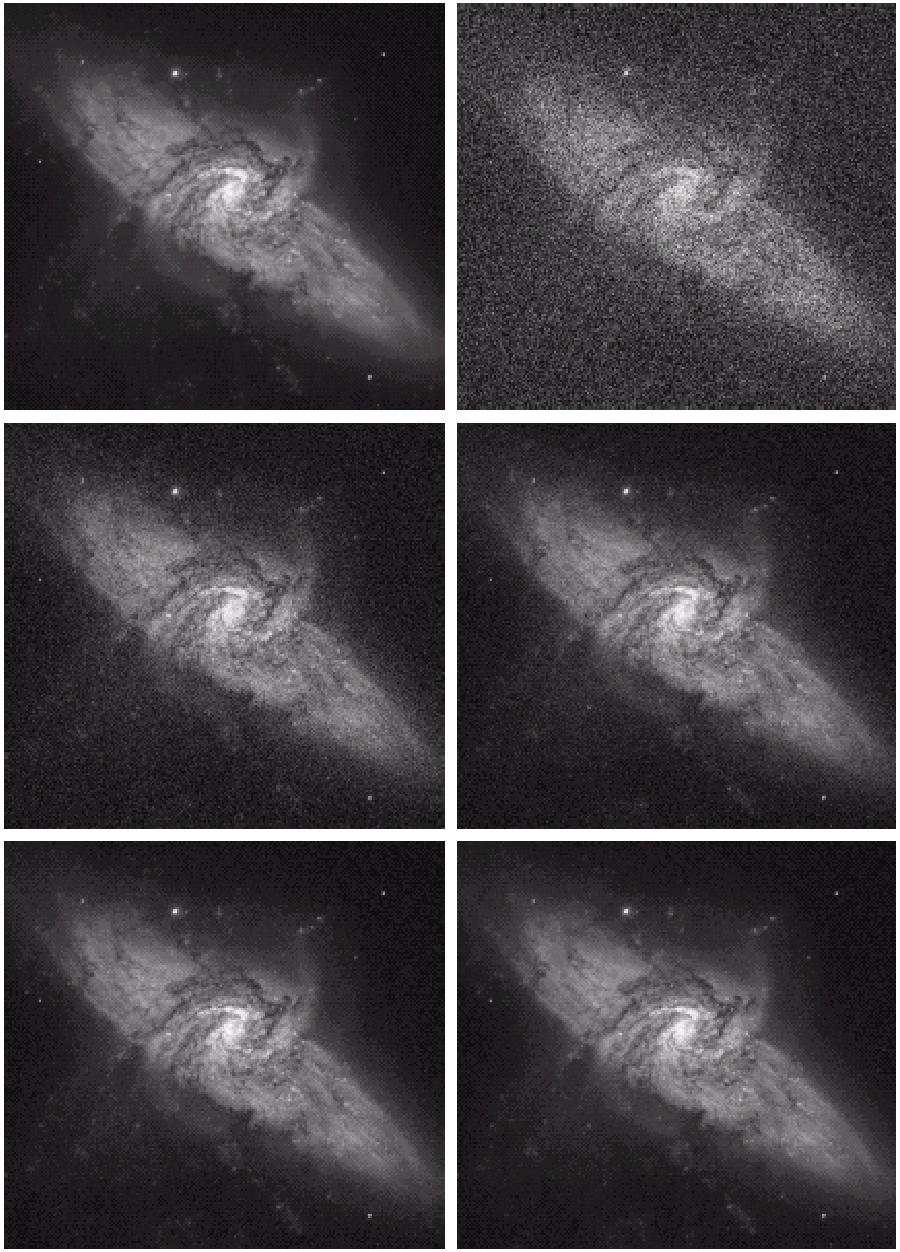


a b

FIGURE 3.29 Enhancement by image subtraction. (a) Mask image. (b) An image (taken after injection of a contrast medium into the bloodstream) with mask subtracted out.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Arithmetic Operation: Image Averaging



Application : Noise reduction

Degraded image

$$g(x,y)=f(x,y)+\eta(x,y)$$

(noise)

Image averaging

$$\bar{g}(x,y)=\frac{1}{K}\sum_{i=1}^{K}g_i(x,y)$$

Taking the average makes the variance of noise diminish

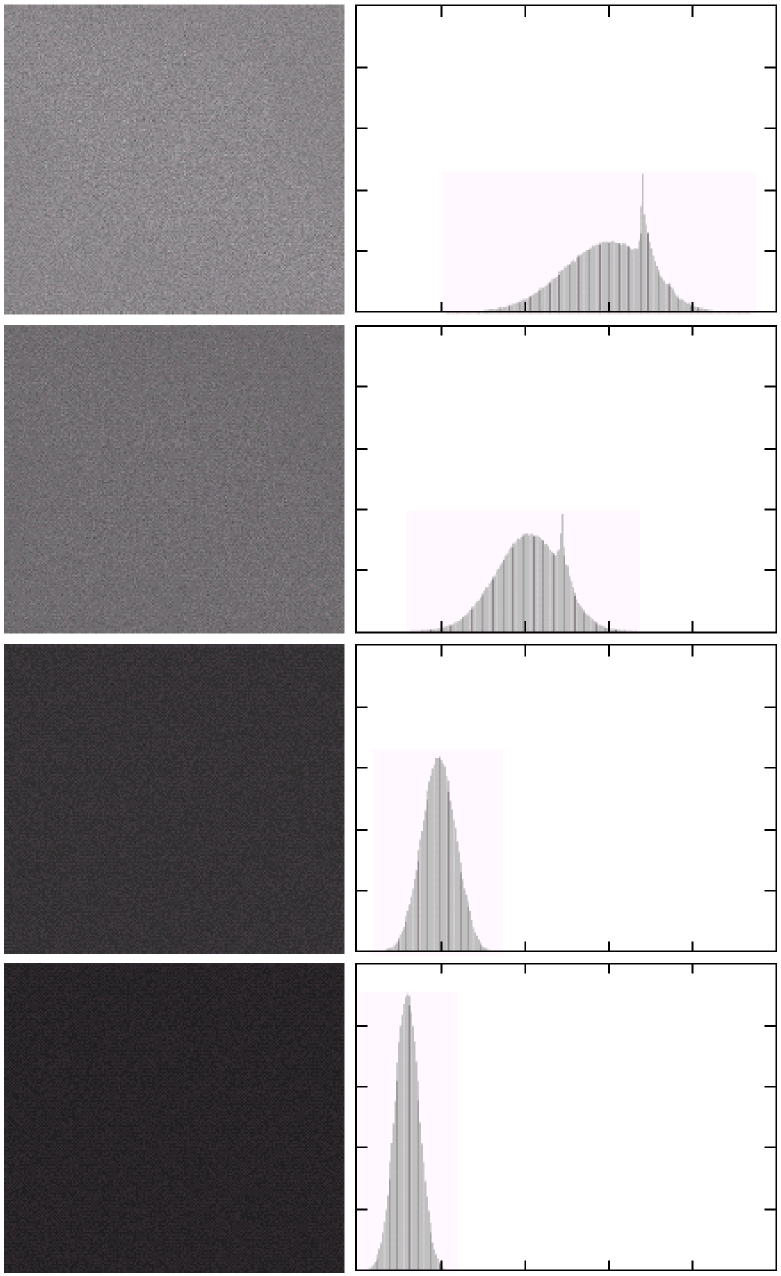
$$\sigma_{\bar{g}(x,y)}=\frac{1}{\sqrt{K}}\sigma_{\eta(x,y)}$$

a	b
c	d
e	f

FIGURE 3.30 (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging $K = 8, 16, 64$, and 128 noisy images. (Original image courtesy of NASA.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Arithmetic Operation: Image Averaging (cont.)



a b

FIGURE 3.31 (a) From top to bottom: Difference images between Fig. 3.30(a) and the four images in Figs. 3.30(c) through (f), respectively. (b) Corresponding histograms.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Sometime we need to manipulate values obtained from neighboring pixels

Example: How can we compute an average value of pixels in a 3x3 region center at a pixel z?

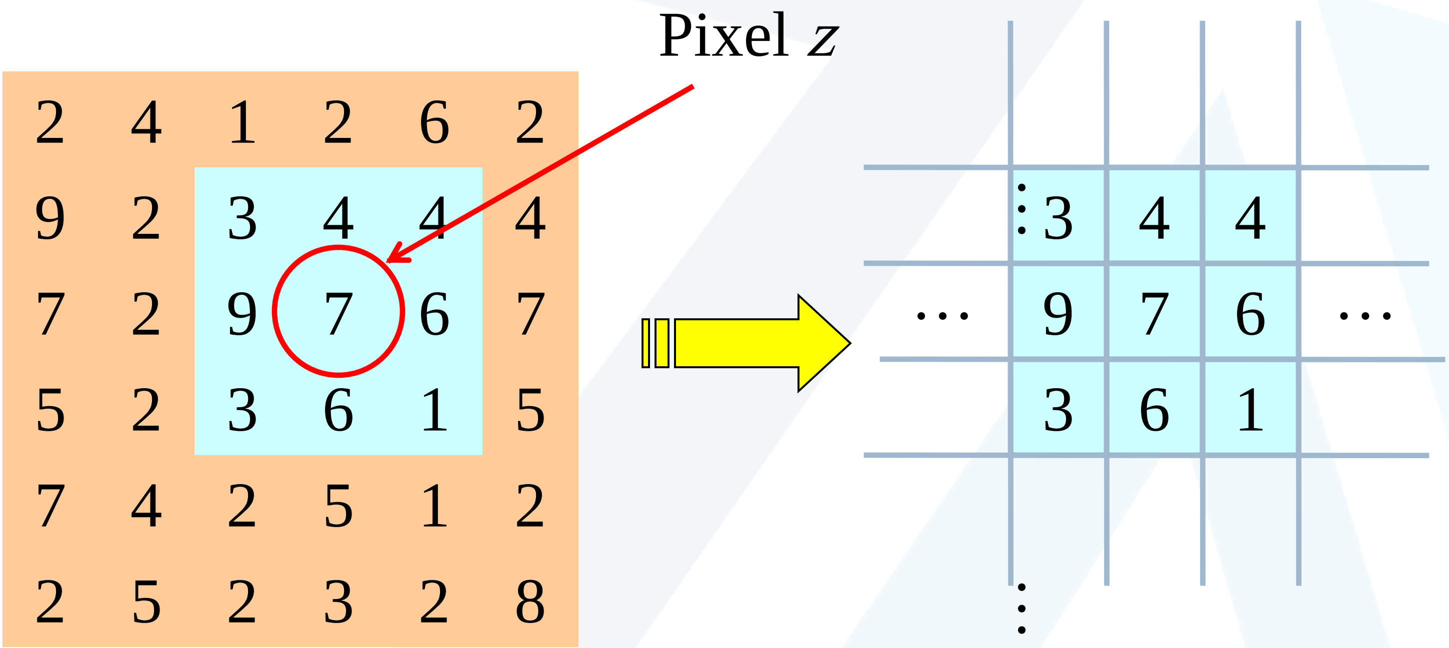
Pixel z

2	4	1	2	6	2
9	2	3	4	4	4
7	2	9	7	6	7
5	2	3	6	1	5
7	4	2	5	1	2
2	5	2	3	2	8

Image

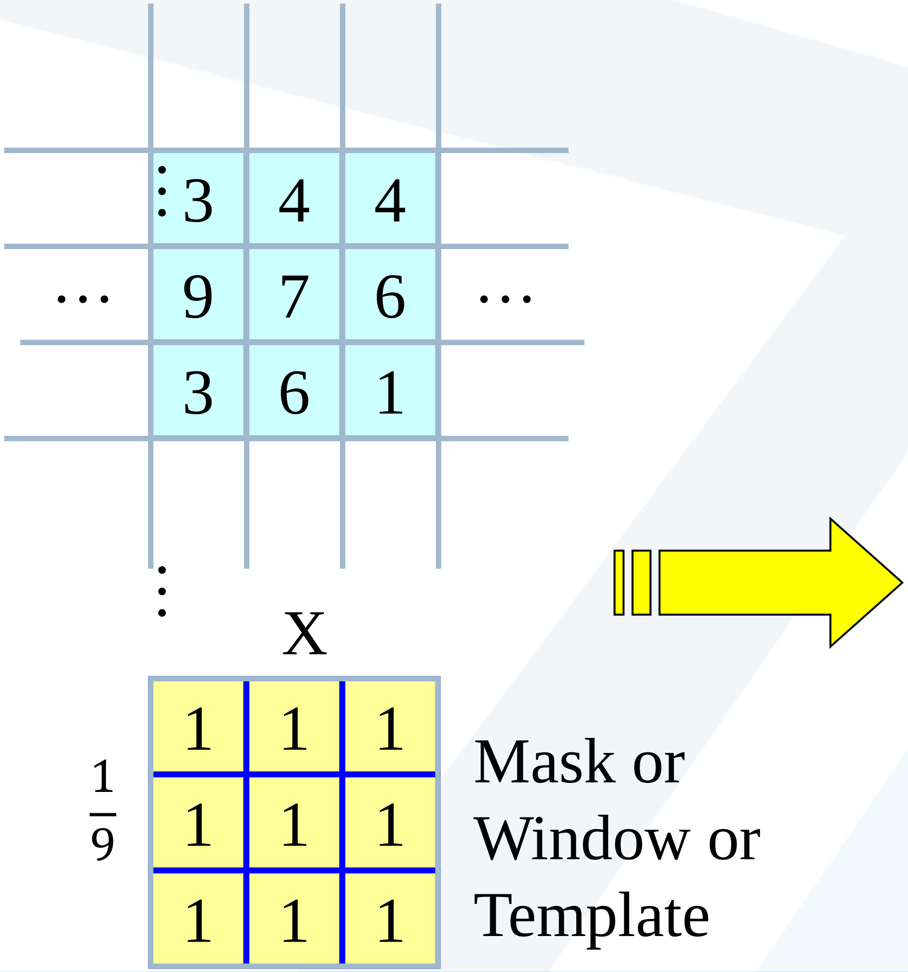
Basics of Spatial Filtering (cont.)

Step 1. Selected only needed pixels



Basics of Spatial Filtering (cont.)

Step 2. Multiply every pixel by 1/9 and then sum up the values



$$y = \frac{1}{9}\cdot 3 + \frac{1}{9}\cdot 4 + \frac{1}{9}\cdot 4 + \frac{1}{9}\cdot 9 + \frac{1}{9}\cdot 7 + \frac{1}{9}\cdot 6 + \frac{1}{9}\cdot 3 + \frac{1}{9}\cdot 6 + \frac{1}{9}\cdot 1$$

Basics of Spatial Filtering (cont.)

Question: How to compute the 3x3 average values at every pixels?

2	4	1	2	6	2
9	2	3	4	4	4
7	2	9	7	6	7
5	2	3	6	1	5
7	4	2	5	1	2

Solution: Imagine that we have a 3x3 window that can be placed everywhere on the image

Masking Window

Basics of Spatial Filtering (cont.)

Step 1: Move the window to the first location where we want to compute the average value and then select only pixels inside the window.

2	4	1	2	6	2
9	2	3	4	4	4
7	2	9	7	6	7
5	2	3	6	1	5
7	4	2	5	1	2

Original image

2	4	1
9	2	3
7	2	9

Sub image *p*

Step 2: Compute the average value

$$y = \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{1}{9} \cdot p(i,j))$$

Step 3: Place the result at the pixel in the output image

	4.3		

Output image

Step 4: Move the window to the next location and go to Step 2

Basics of Spatial Filtering (cont.)

The 3x3 averaging method is one example of the *mask operation* or *Spatial filtering*.

- The mask operation has the corresponding mask (sometimes called window or template).
- The mask contains coefficients to be multiplied with pixel values.

w(1,1)	w(2,1)	w(3,1)
w(1,2)	w(2,2)	w(3,2)
w(3,1)	w(3,2)	w(3,3)

Mask coefficients

Example : moving averaging

$\frac{1}{9}$

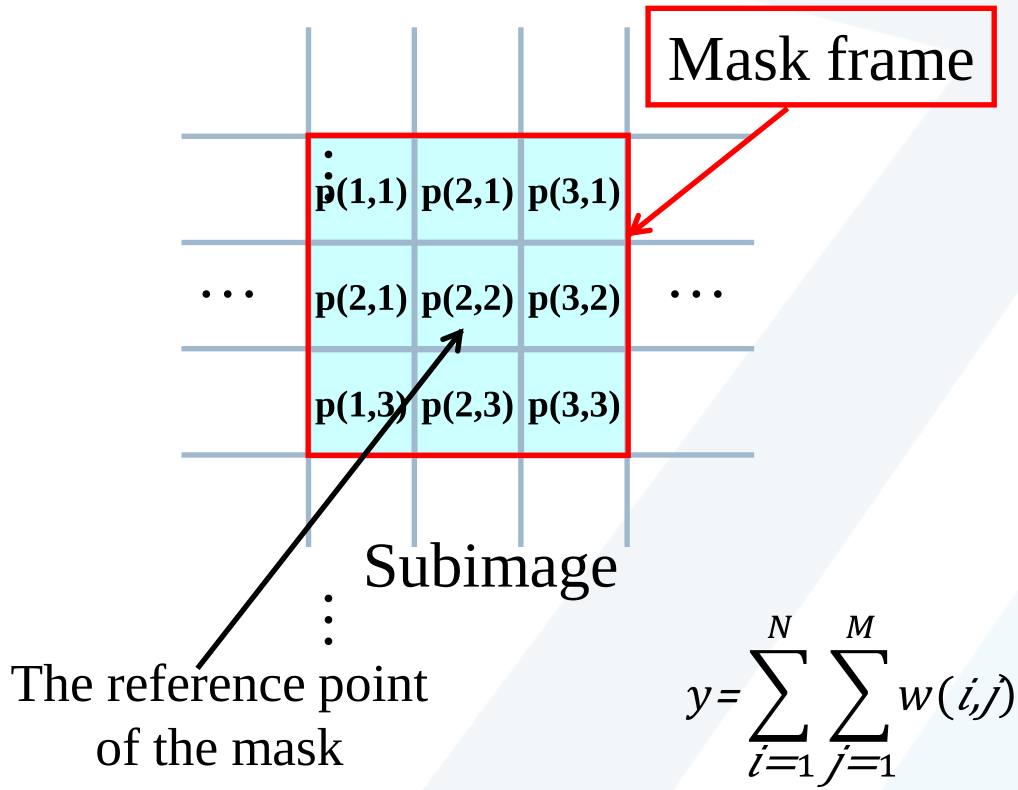
1	1	1
1	1	1
1	1	1

The mask of the 3x3 moving average filter has all coefficients = 1/9

Basics of Spatial Filtering (cont.)

The mask operation at each point is performed by:

1. Move the reference point (center) of mask to the location to be computed
2. Compute *sum of products* between mask coefficients and pixels in subimage under the mask.



w(1,1)	w(2,1)	w(3,1)
w(1,2)	w(2,2)	w(3,2)
w(3,1)	w(3,2)	w(3,3)

Mask coefficients

$$y=\sum_{i=1}^{N}\sum_{j=1}^{M} w(i,j)\cdot p(i,j)$$

Basics of Spatial Filtering (cont.)

The spatial filtering on the whole image is given by:

1. Move the mask over the image at each location.
1. Compute sum of products between the mask coefficeints and pixels inside subimage under the mask.
3. Store the results at the corresponding pixels of the output image.
4. Move the mask to the next location and go to step 2 until all pixel locations have been used.

Examples of Spatial Filtering Masks

Examples of the masks

Sobel operators

-1	0	1
-2	0	2
-1	0	1

gradient $at\, x = \frac{\partial P}{\partial x}$

-1	-2	-1
0	0	0
1	2	1

gradient $at\, y = \frac{\partial P}{\partial y}$

3x3 moving average filter

$\frac{1}{9}$

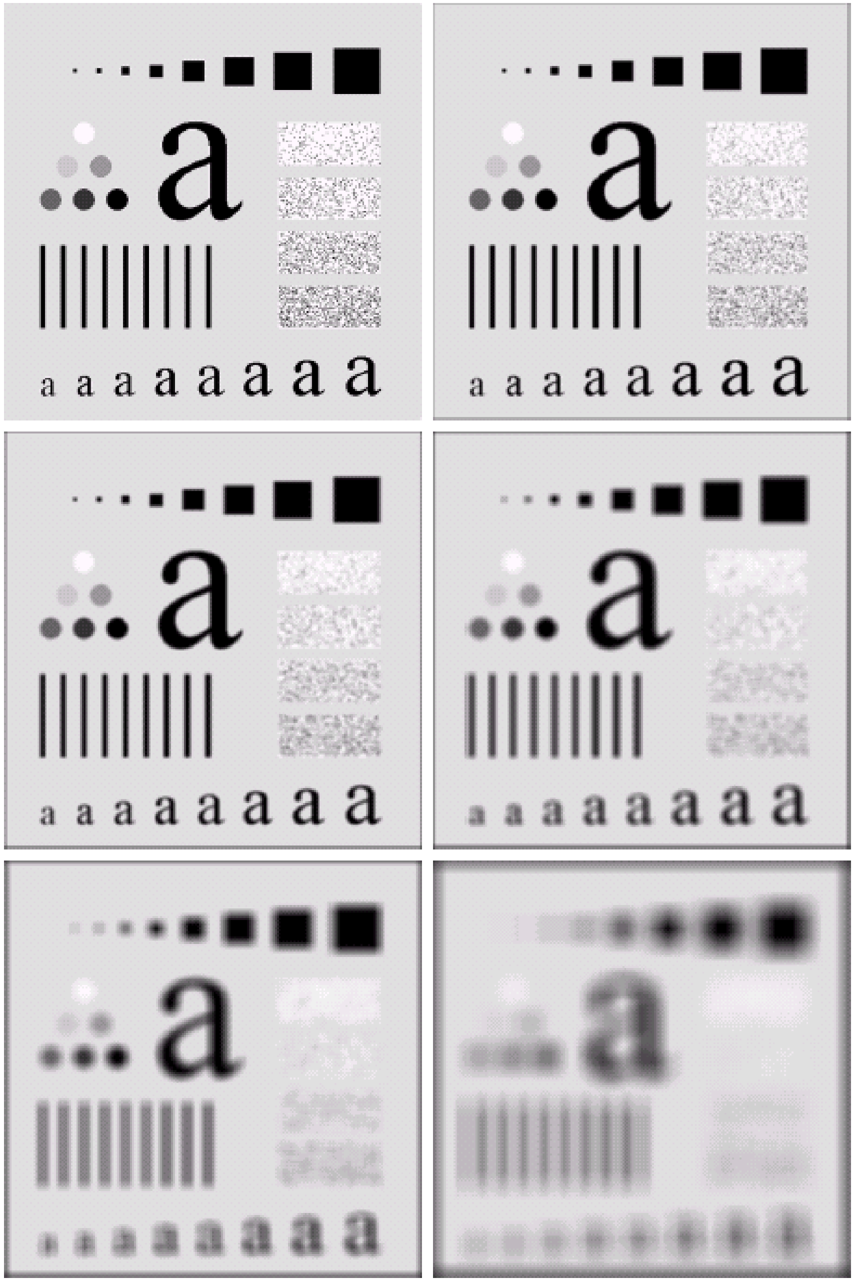
1	1	1
1	1	1
1	1	1

3x3 sharpening filter

$\frac{1}{9}$

-1	-1	-1
-1	8	-1
-1	-1	-1

Smoothing Linear Filter : Moving Average



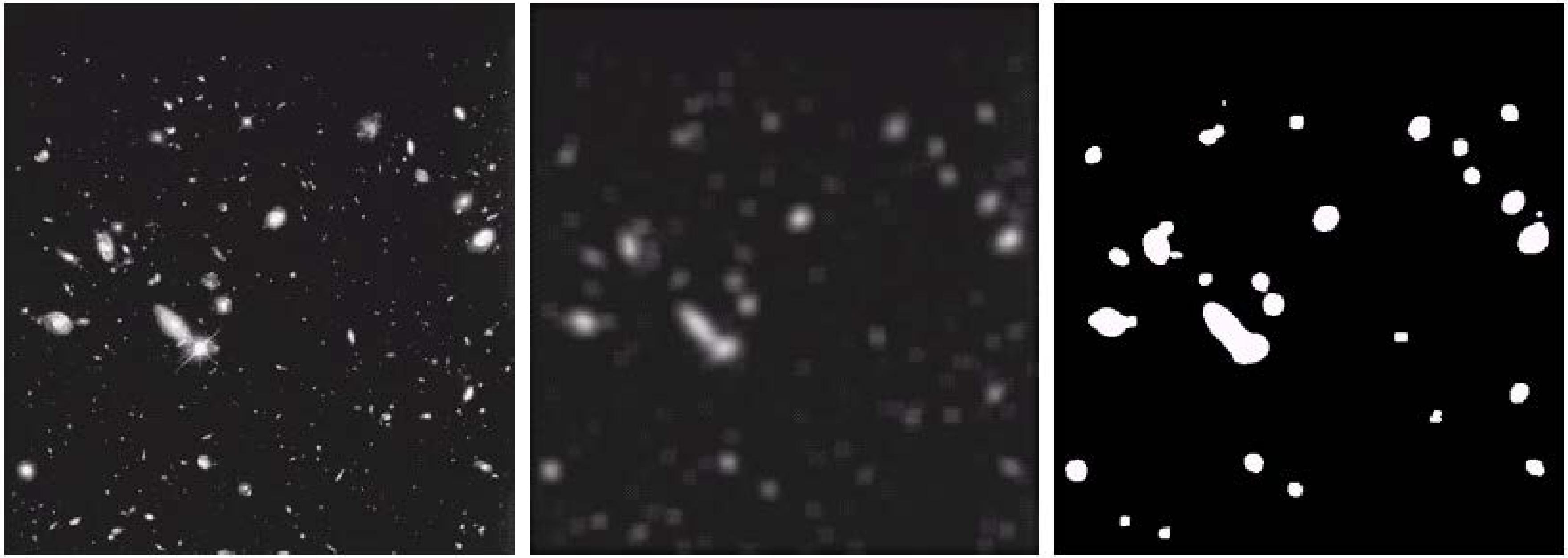
Application : noise reduction and image smoothing

Disadvantage: lose sharp details

a	b
c	d
e	f

FIGURE 3.35 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

Smoothing Linear Filter (cont.)

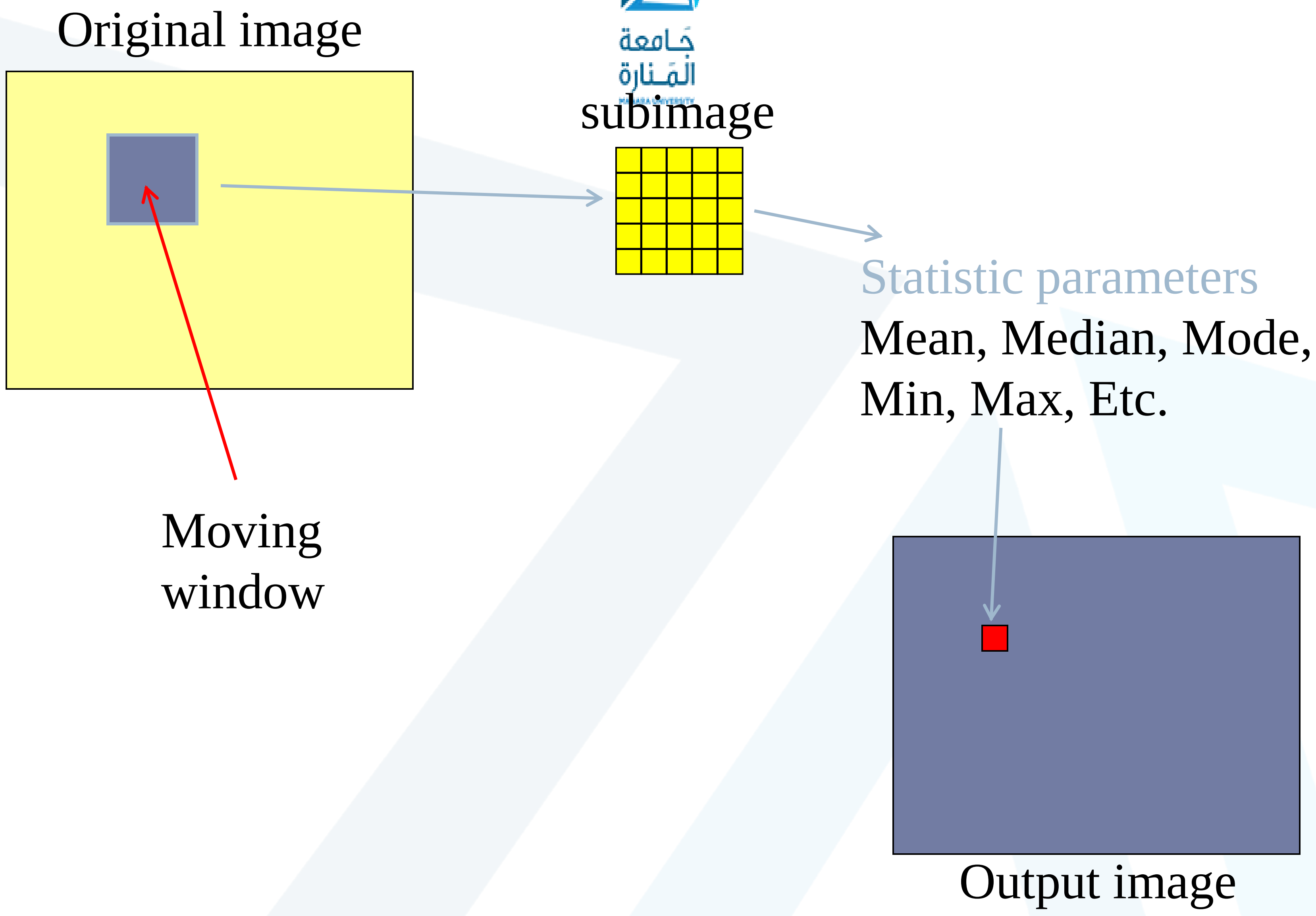


a b c

FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

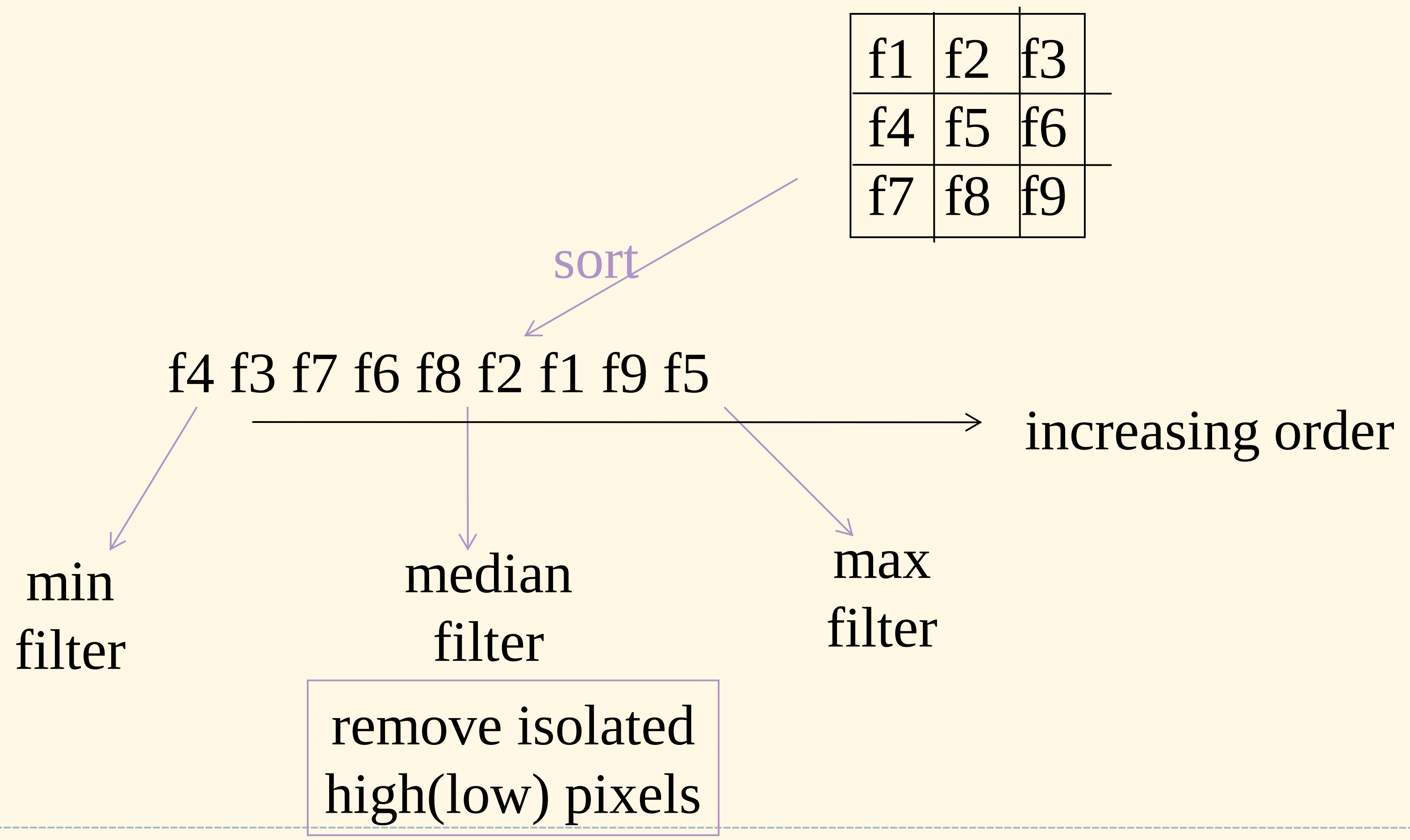
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Order-Statistic Filters

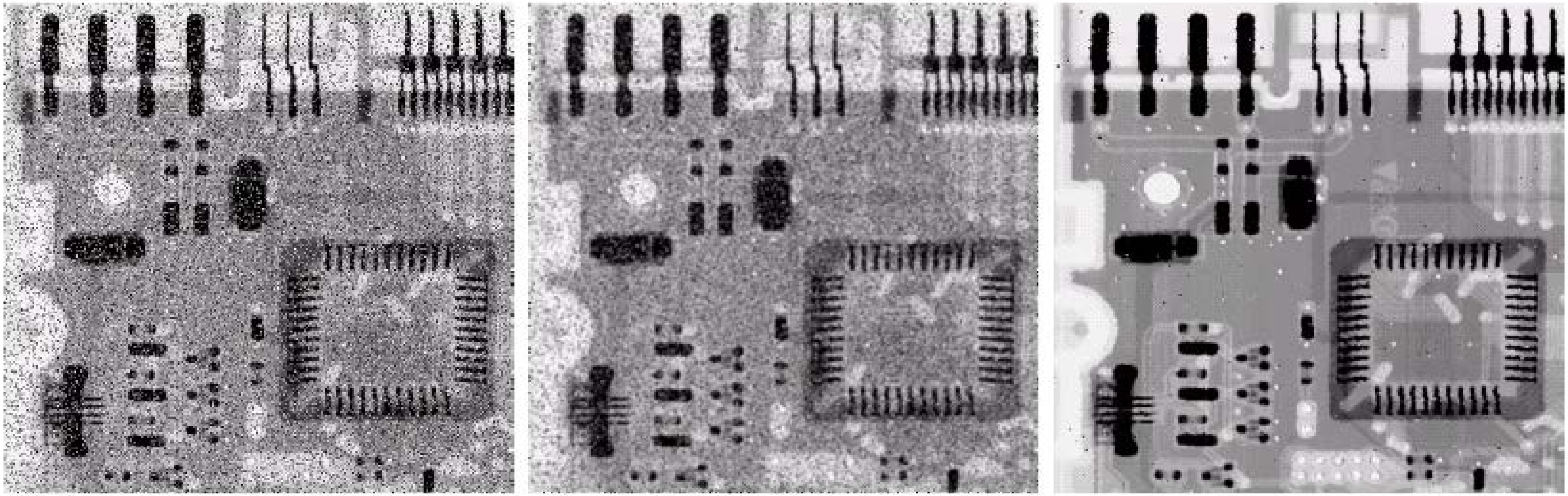


Order-statistics filters

- Pixel neighborhood



Order-Statistic Filters: Median Filter



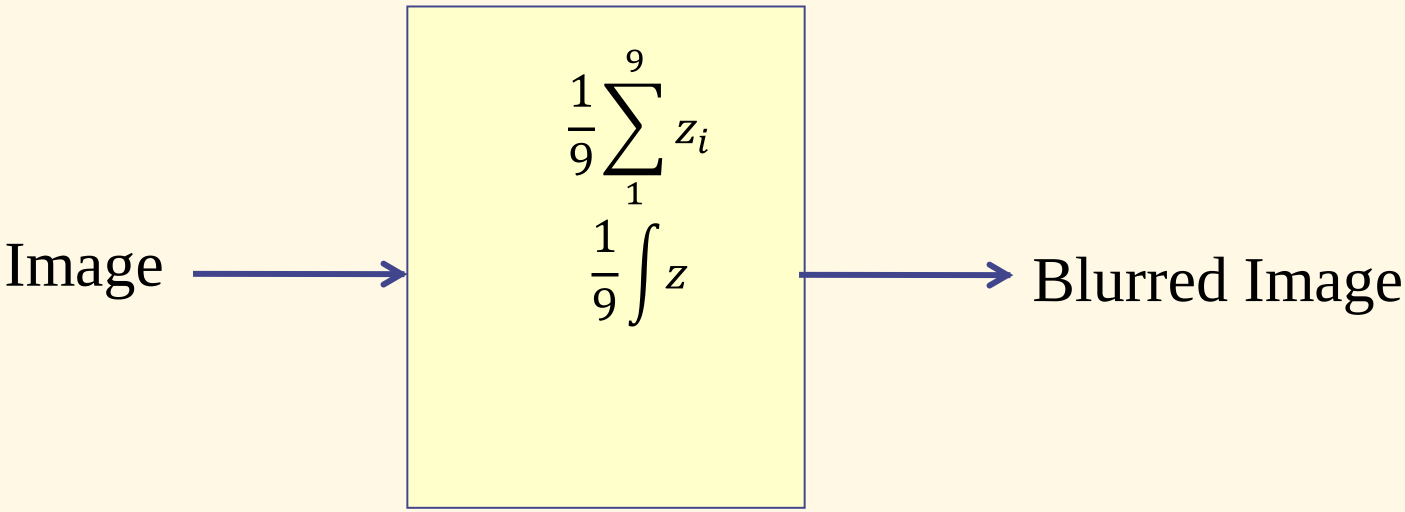
a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Sharpening Spatial Filters

The principal objective of sharpening is to highlight fine detail in an image or to enhance detail that has been blurred.



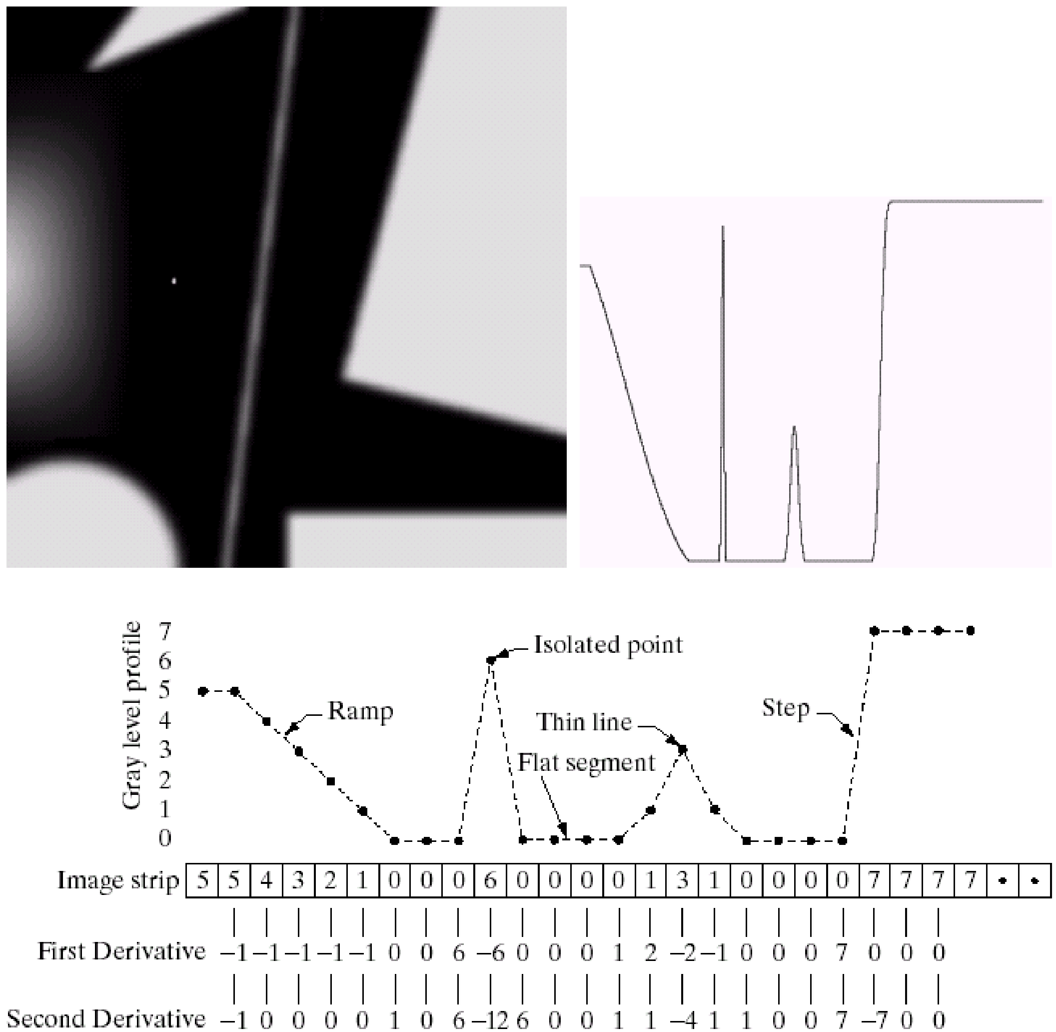
The derivatives of a digital function are defined in terms of differences.

Sharpening Spatial Filters

There are intensity discontinuities near object edges in an image

a b
c

FIGURE 3.38
(a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point. (c) Simplified profile (the points are joined by dashed lines to simplify interpretation).



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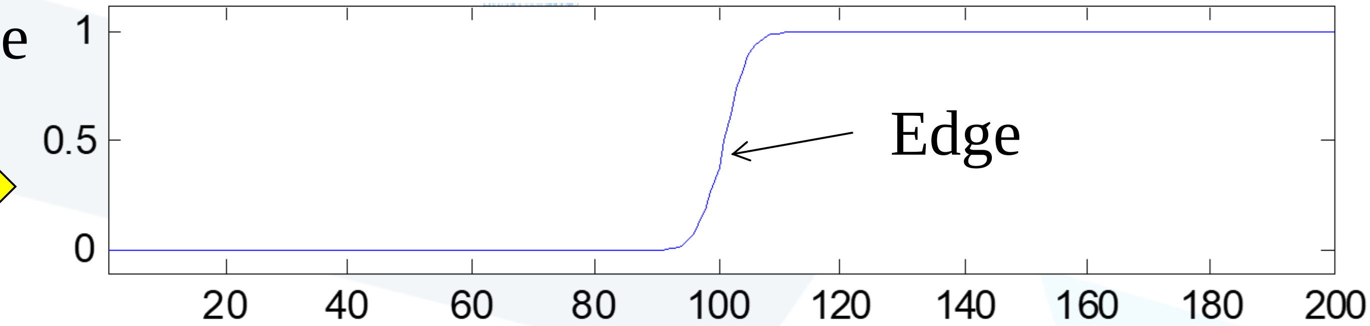
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Laplacian Sharpening : How it works

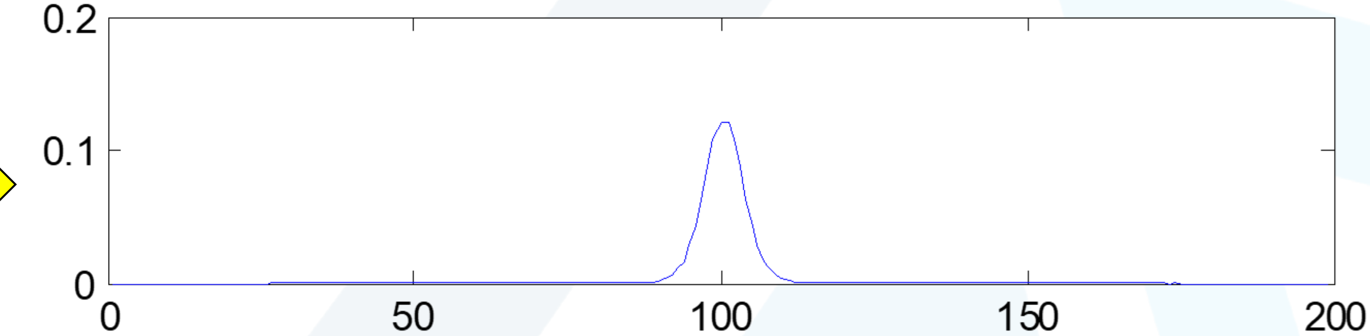
Intensity profile

$p(x)$



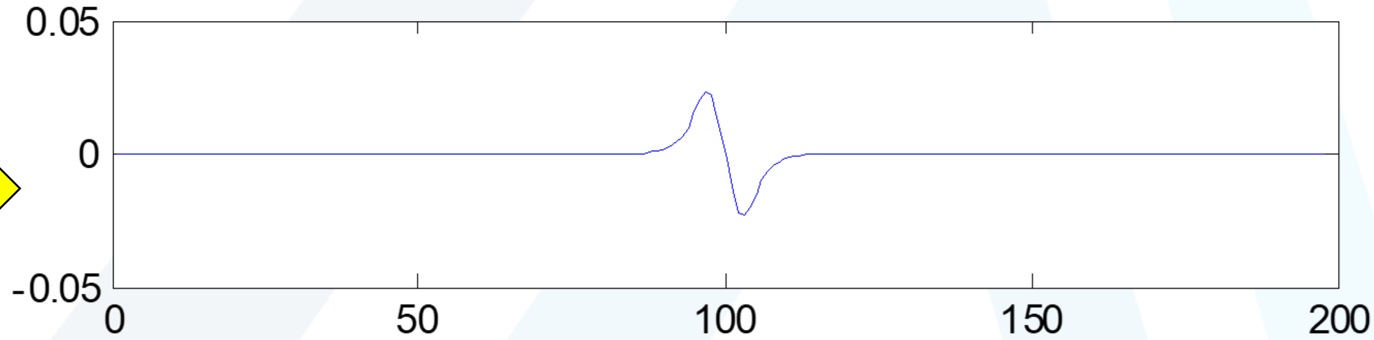
1st derivative

$$\frac{dp}{dx}$$



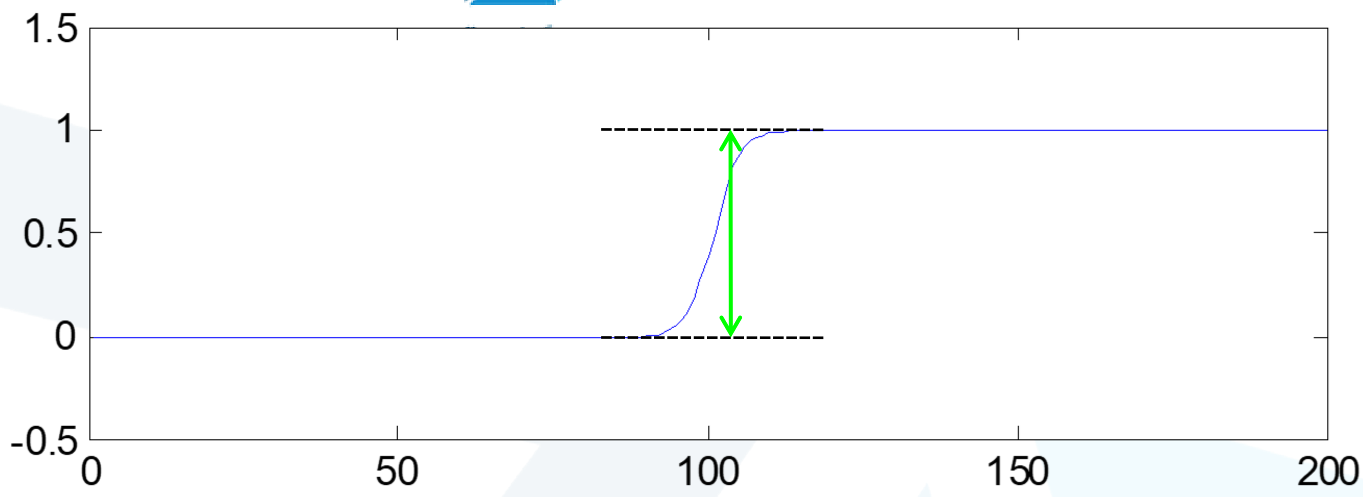
2nd derivative

$$\frac{d^2p}{dx^2}$$

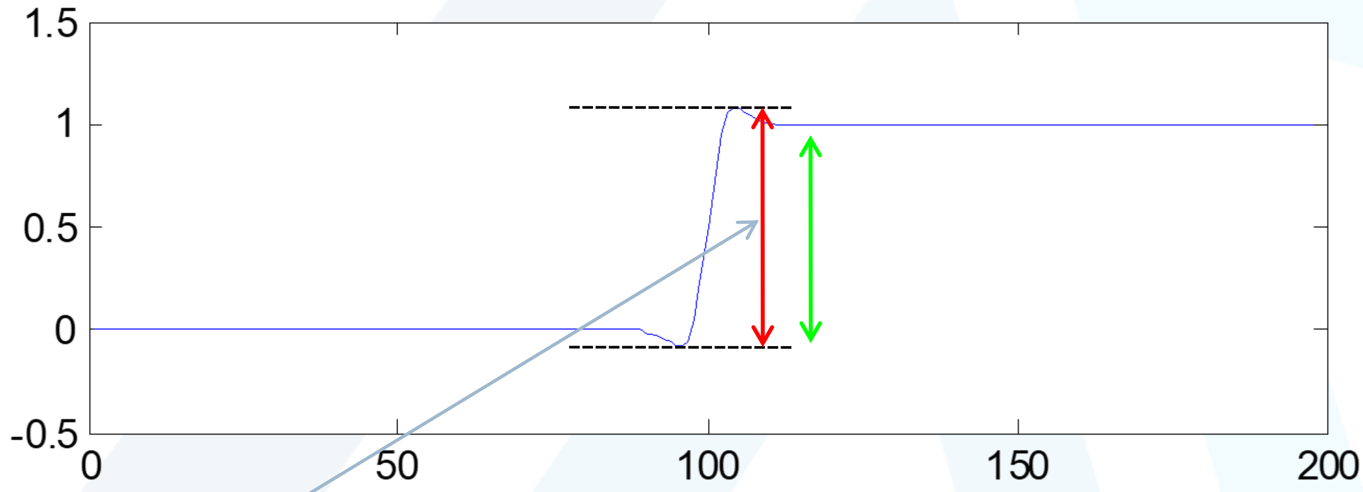


Laplacian Sharpening : How it works (cont.)

$p(x)$

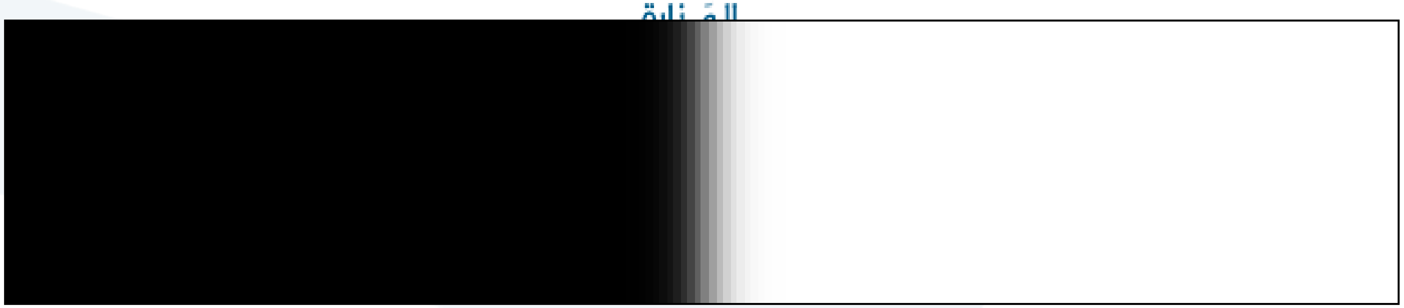


$$p(x) - 10\frac{d^2p}{dx^2}$$



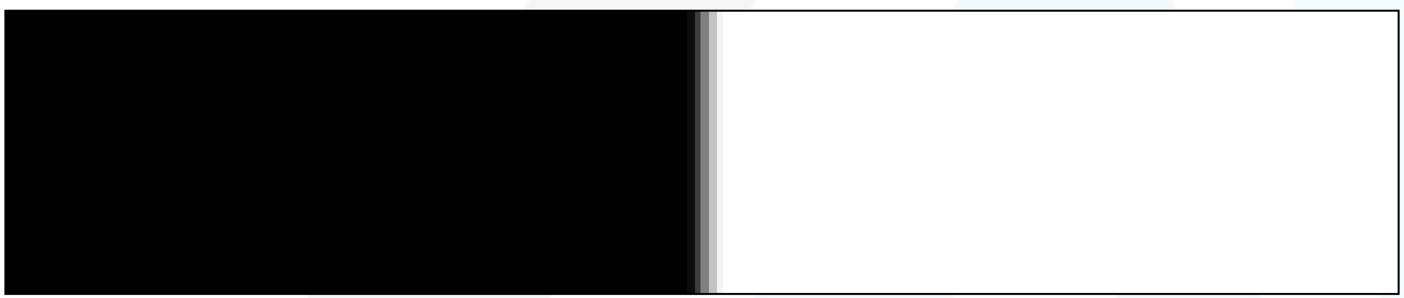
Laplacian sharpening results in larger intensity discontinuity near the edge.

Laplacian Sharpening : How it works (cont.)



Before sharpening

$$p(x)$$



After sharpening

$$p(x) - 10\frac{d^2p}{dx^2}$$

Laplacian Masks

جامعة المنارة

Used for estimating image Laplacian

$$\nabla^2 P = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}$$

-1	-1	-1
-1	8	-1
-1	-1	-1

0	-1	0
-1	4	-1
0	-1	0

The center of the mask is positive

or

1	1	1
1	-8	1
1	1	1

0	1	0
1	-4	1
0	1	0

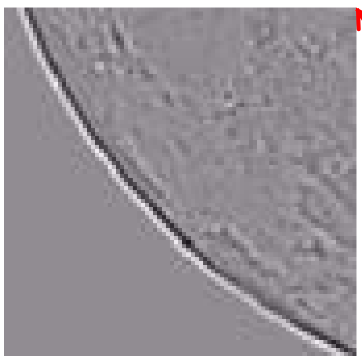
The center of the mask is negative

Application: Enhance edge, line, point

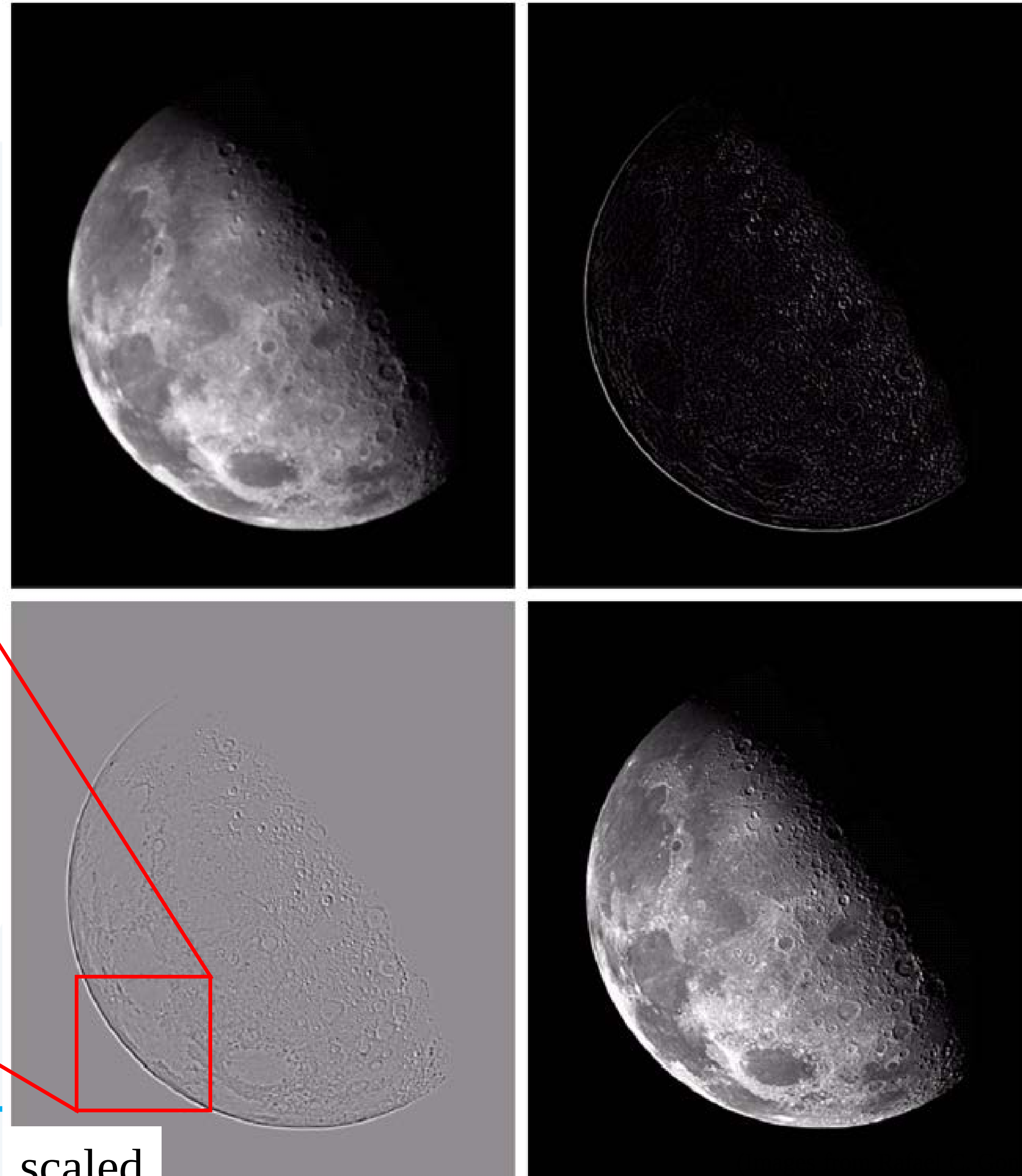
Disadvantage: Enhance noise

Laplacian Sharpening Example

p



$\nabla^2 P$



$\nabla^2 P$

$P - \nabla^2 P$

...alez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Laplacian Sharpening (cont.)

Mask for $P - \nabla^2 P$

0	-1	0
-1	5	-1
0	-1	0

or

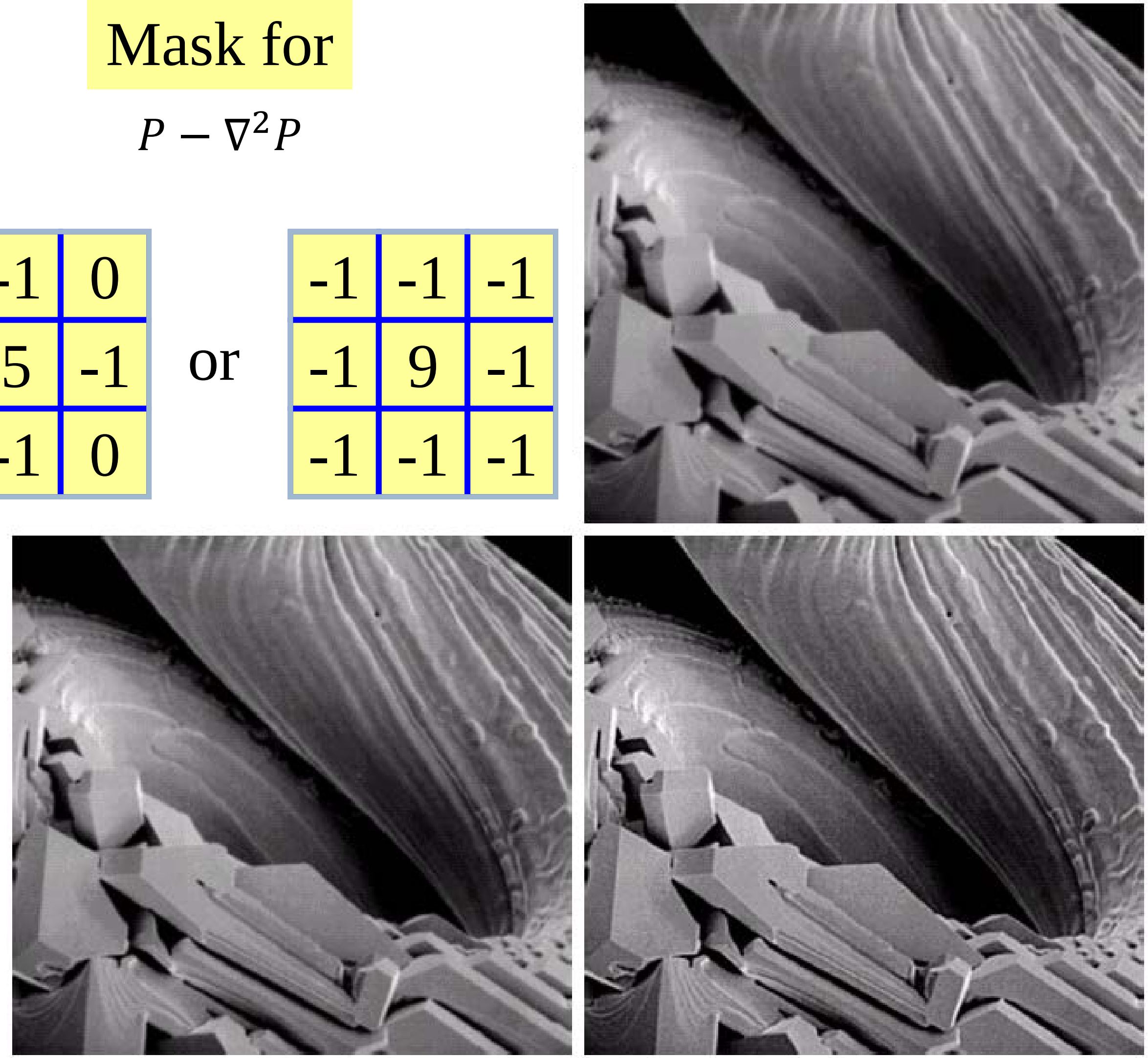
-1	-1	-1
-1	9	-1
-1	-1	-1

Mask for $\nabla^2 P$

1	1	1
1	-8	1
1	1	1

or

0	1	0
1	-4	1
0	1	0



a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Unsharp Masking and High-Boost Filtering

-1	-1	-1
-1	k+8	-1
-1	-1	-1

0	-1	0
-1	k+4	-1
0	-1	0

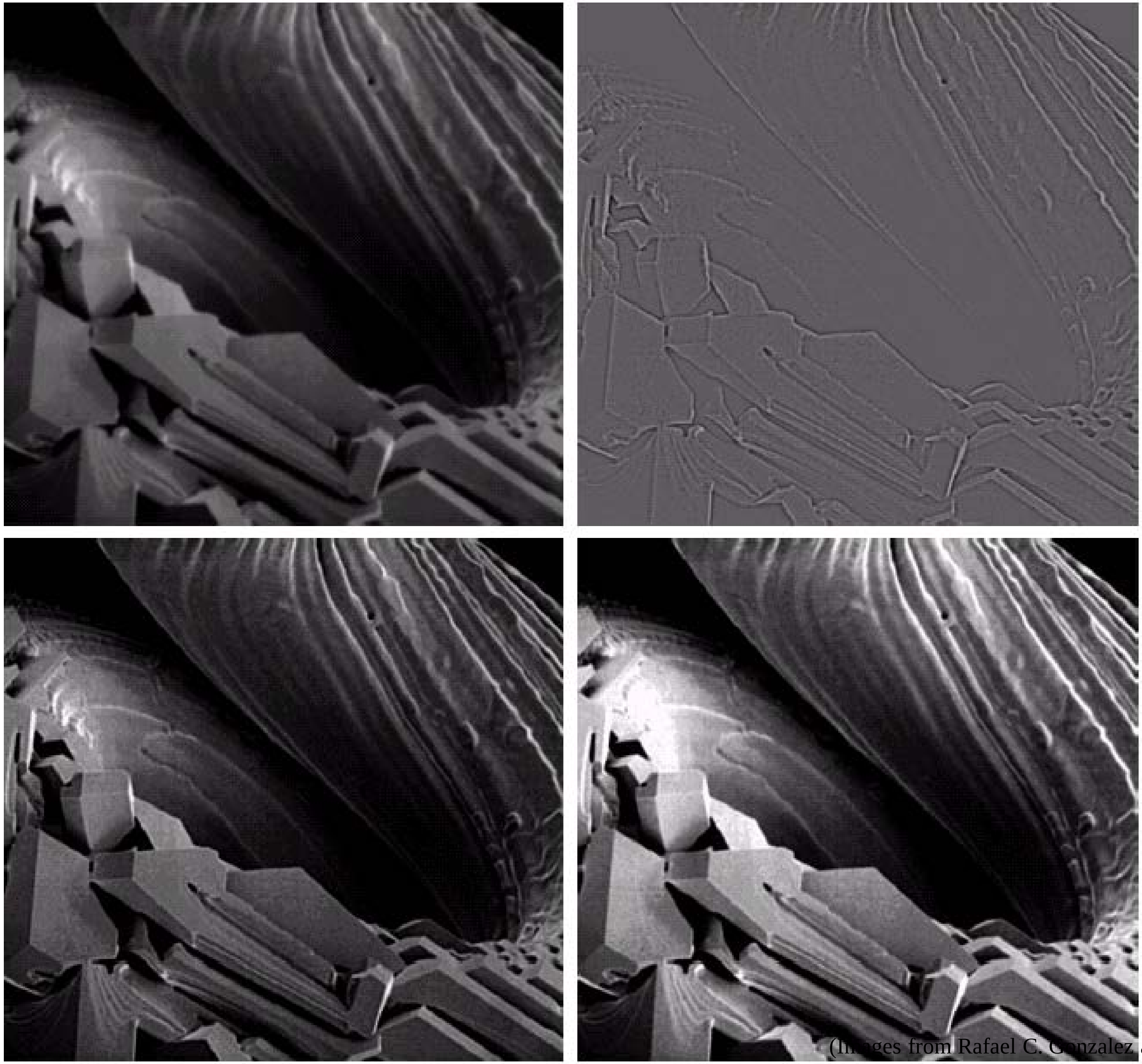
Equation:

$$P_{hb}(x,y) = \begin{cases} kP(x,y) - \nabla^2 P(x,y) \\ kP(x,y) + \nabla^2 P(x,y) \end{cases}$$

→ The center of the mask is negative

→ The center of the mask is positive

Unsharp Masking and High-Boost Filtering (cont.)



a b
c d

FIGURE 3.43 (a) Same as Fig. 3.41(c), but darker. (a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$. (c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

First Order Derivative

Intensity profile

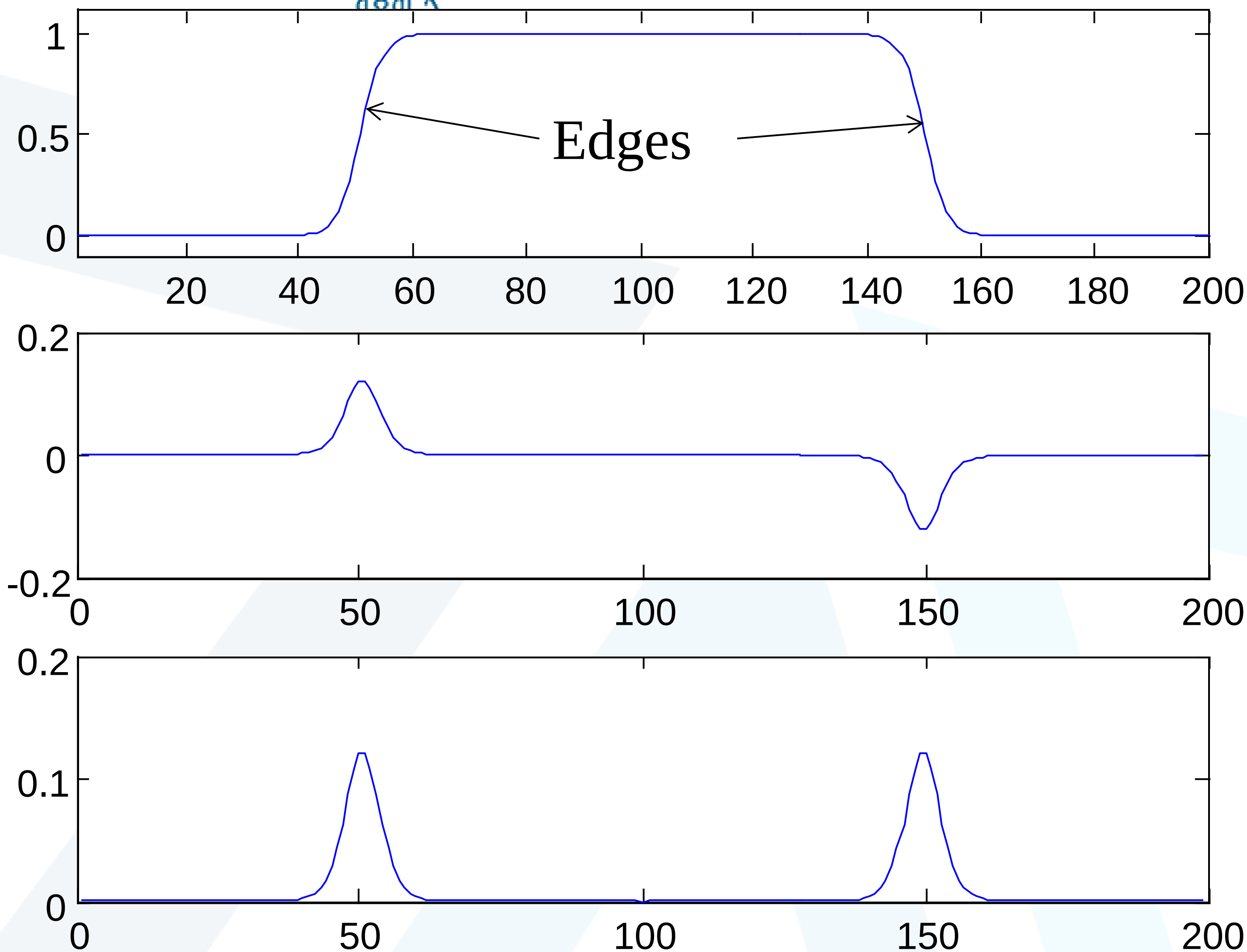
$p(x)$

1st derivative

$$\frac{dp}{dx}$$

2nd derivative

$$\left|\frac{dp}{dx}\right|$$



First Order Partial Derivative:

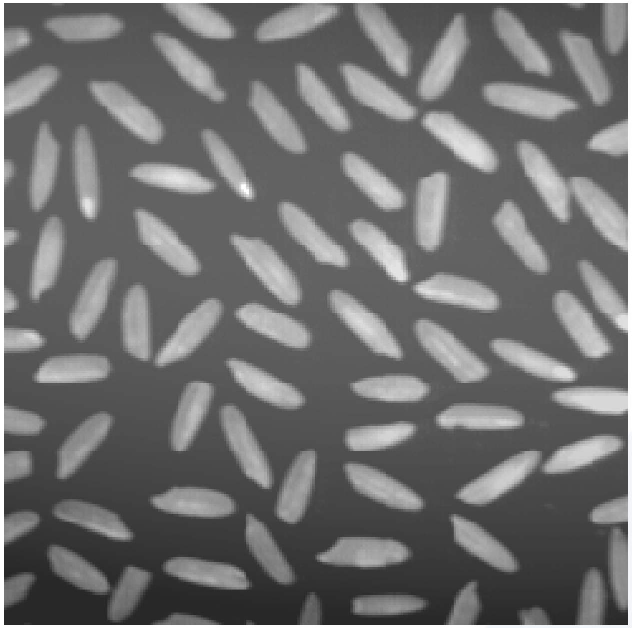
Sobel operators

-1	0	1
-2	0	2
-1	0	1

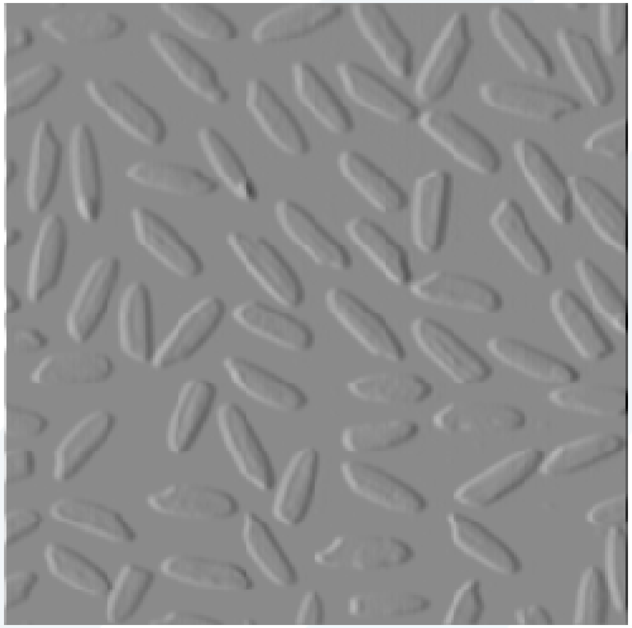
to *compute* $\frac{\partial P}{\partial x}$

-1	-2	-1
0	0	0
1	2	1

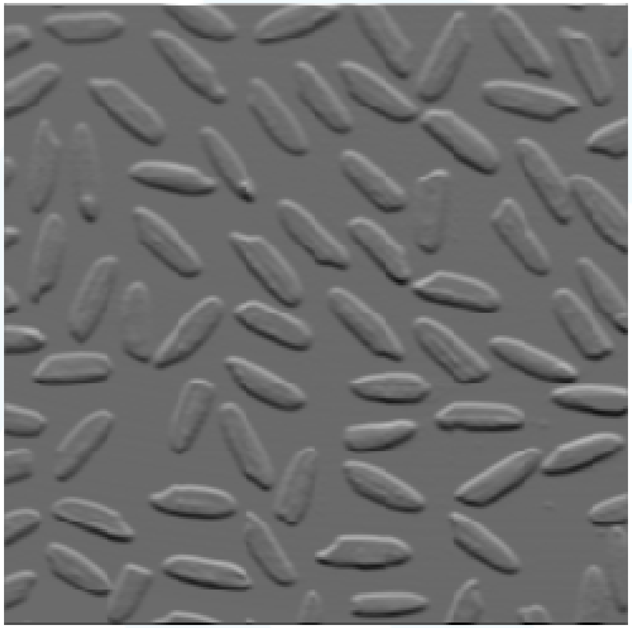
to *compute* $\frac{\partial P}{\partial y}$



P



$\frac{\partial P}{\partial x}$



$\frac{\partial P}{\partial y}$

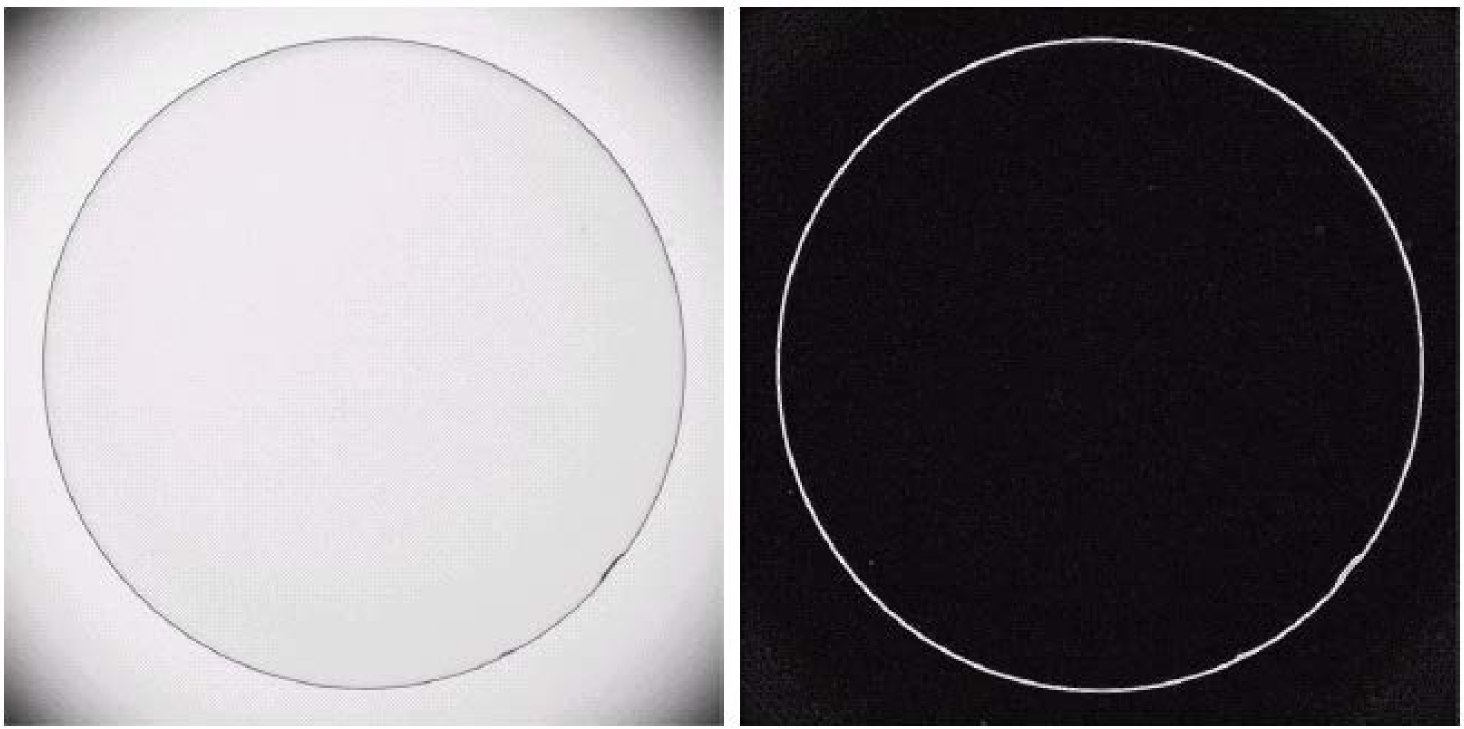
First Order Partial Derivative: Image Gradient

Gradient magnitude

$$|\nabla P| = \sqrt{\left(\frac{\partial P}{\partial x}\right)^2 + \left(\frac{\partial P}{\partial y}\right)^2}$$

Often approximated by

$$|\nabla P| = max[|P(x,y)-P(x+1,y)|, |P(x,y)-P(x,y+1)|]$$

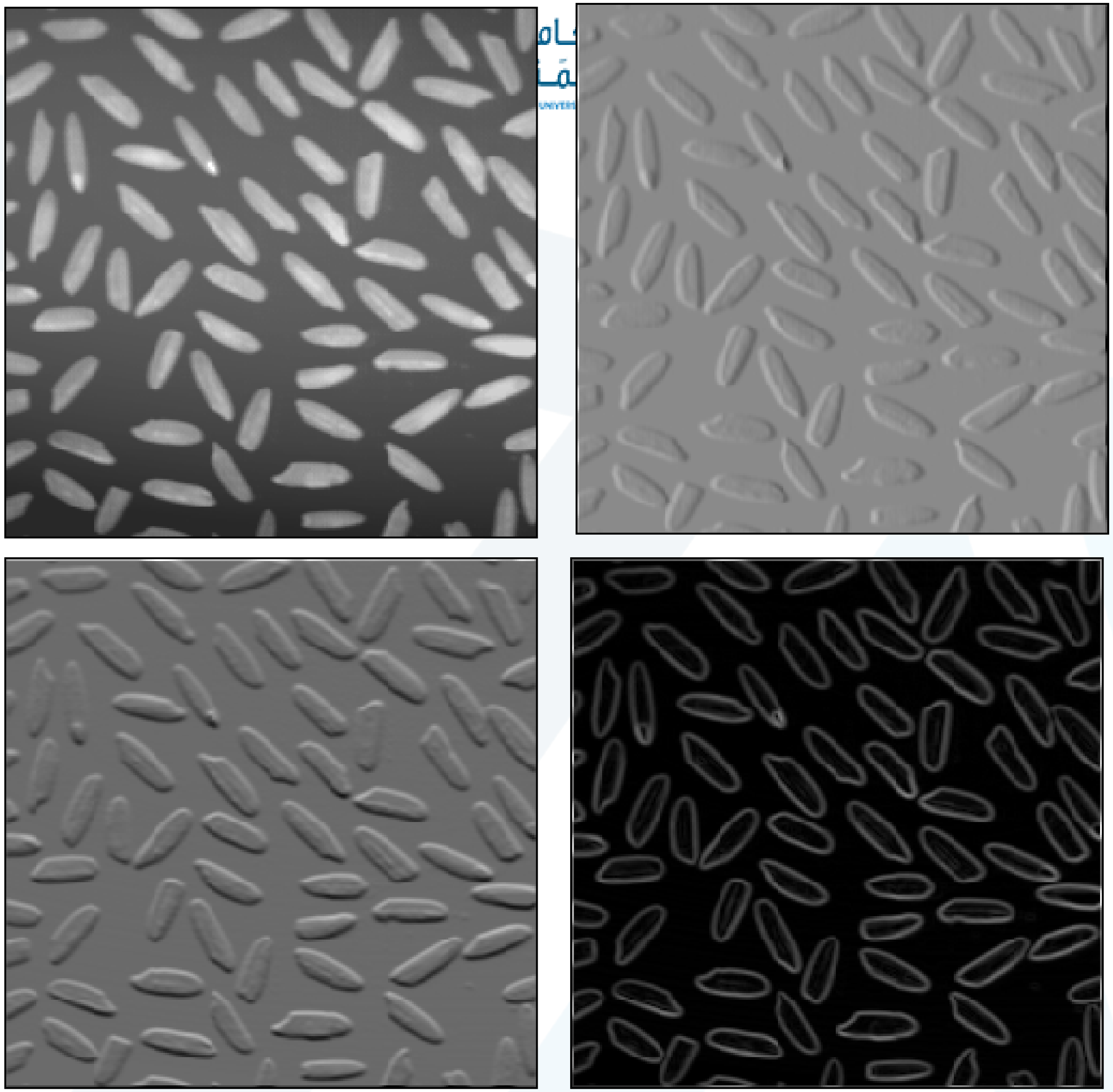


a b

FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

A gradient image emphasizes edges

First Order Partial Derivative: Image Gradient



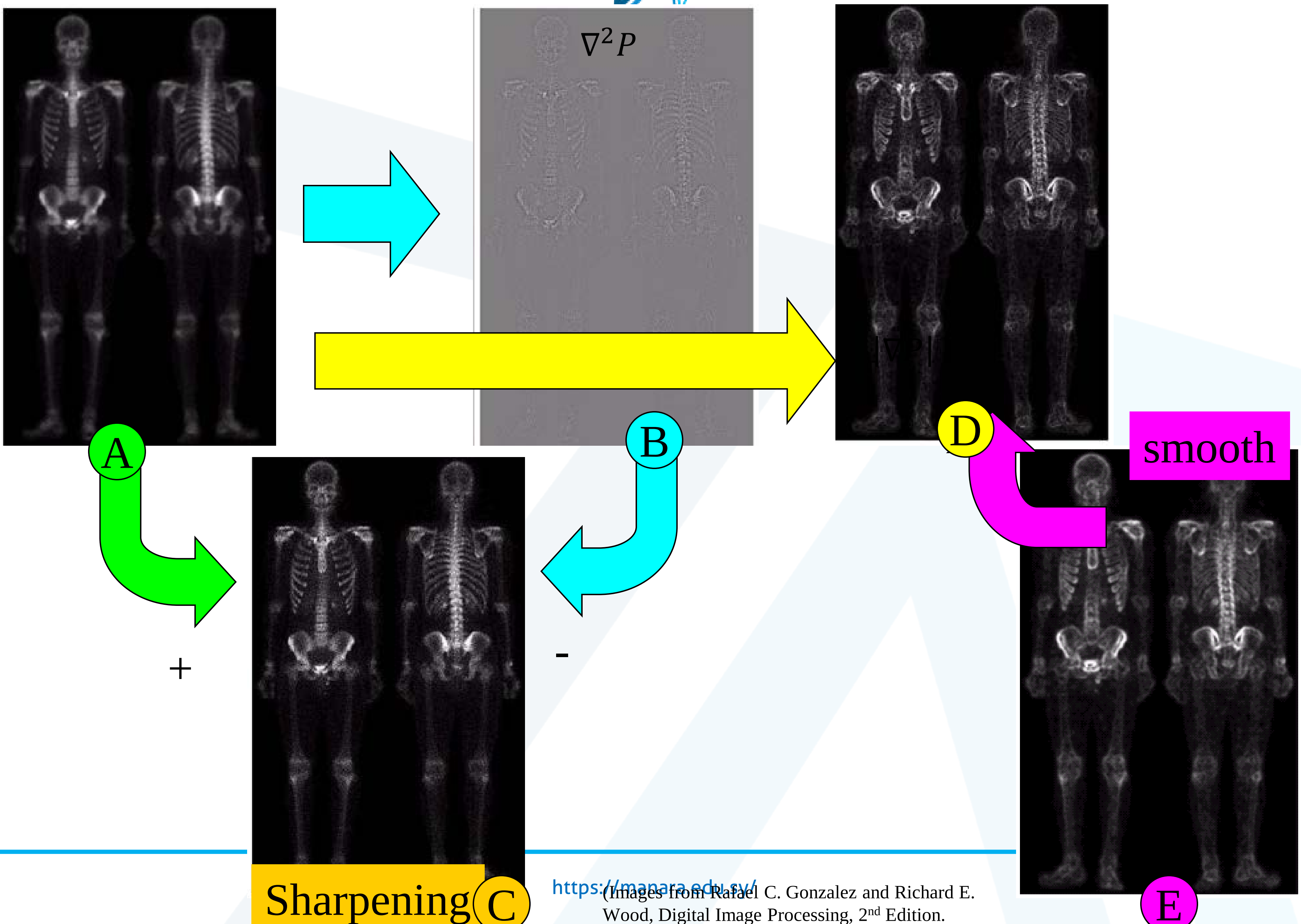
P

$\frac{\partial P}{\partial y}$

$\frac{\partial P}{\partial x}$

$|\nabla P|$

Image Enhancement in the Spatial Domain : Mix things up !



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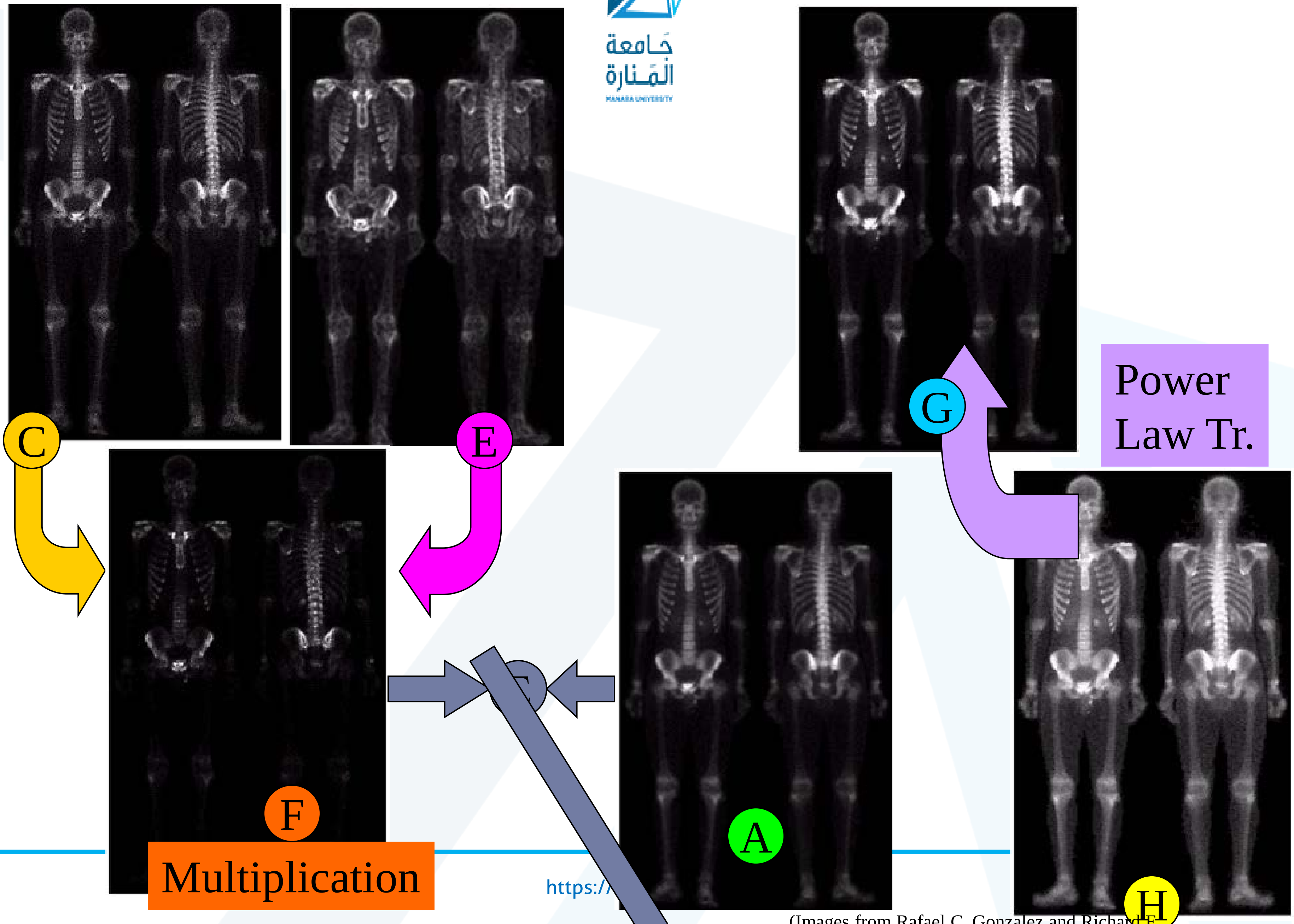
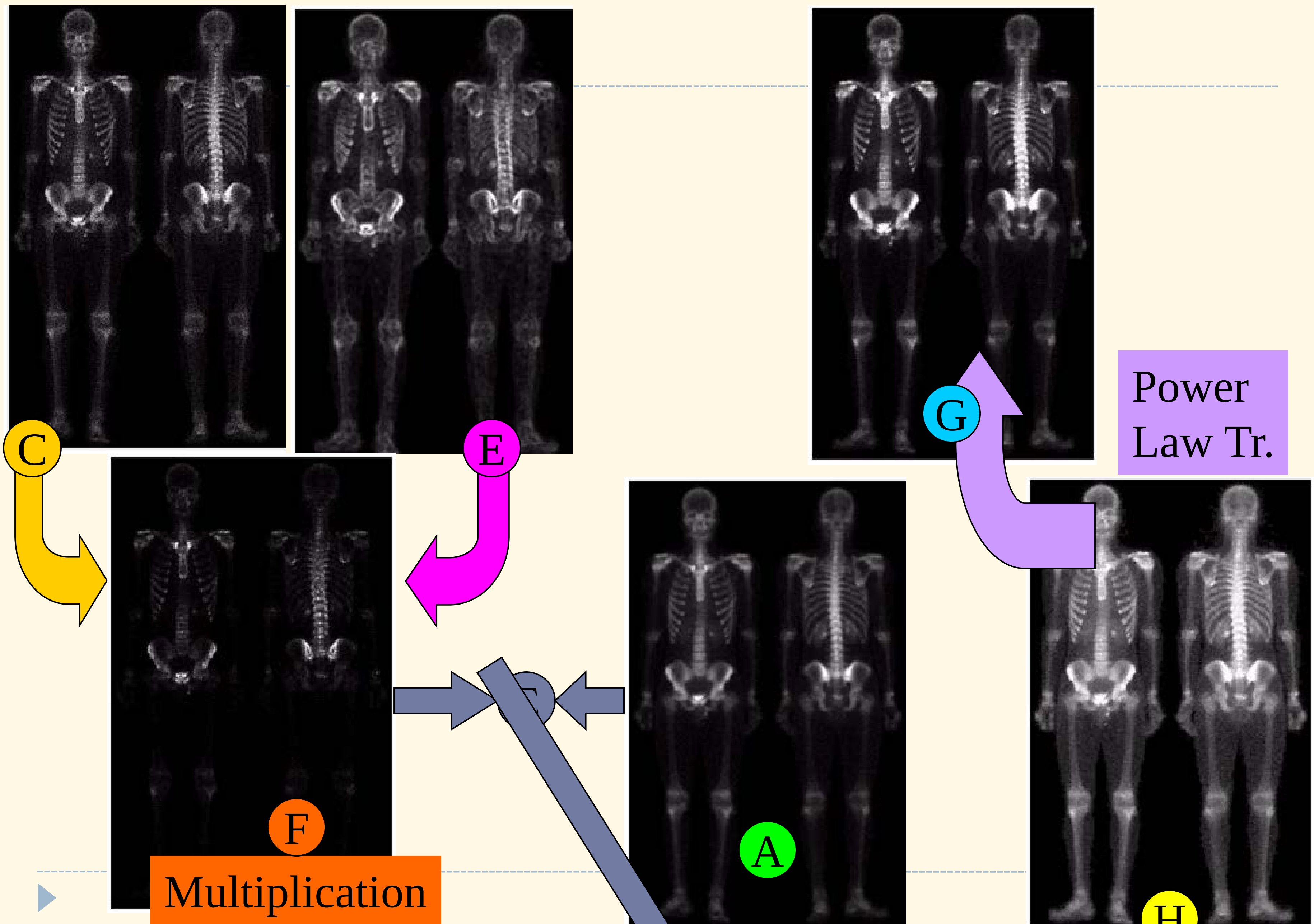
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Image Enhancement in the Spatial Domain : Mix things up !



(Images from Rafael C. Gonzalez and Richard E.

Image Enhancement in the Spatial Domain : *Mix things up !*



(Images from Rafael C. Gonzalez and Richard E.

Questions…